KD EDUCATION ACADEMY [9582701166]

Time : 6 hour | STD 9 Maths | Total Marks : 230

kd 90+ questions ch-9 Circle

*** Choose the right answer from the given options. [1 Marks Each] [100]**

1. Two point on a circle makes the:

(A) Secant (B) Chord (C) Diameters (D) Diameter

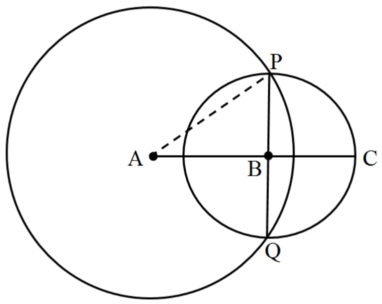
Ans. :

b. Chord

Solution:

A chord is the line joining any two points on the circle.

2. In the given figure, A and B are the centres of two circles having radii 5cm and 3cm respectively and intersecting at points P and Q respectively. If AB = 4cm, then the length of common chord PQ is:



(A) 3cm (B) 7.5cm (C) 9cm (D) 6cm

Ans. :

d. 6cm

Solution:

We know that the line joining their centres is the perpendicular bisector of the common chord.

Join AP.

Then AP = 5cm; AB = 4cm

Also, $AP^2 = BP^2 + AB^2$ [using pythagoras theorem]

$\Rightarrow BP^2 = AP^2 - AB^2$

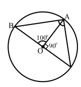
$\Rightarrow BP^2 = 5^2 - 4^2$

$\Rightarrow$ BP = 3cm

$\therefore$ triangle ABP is a right angled and PQ = 2 × BP = (2 × 3)cm = 6cm

3. In the given figure, O is the centre of a circle. If $\angle\mathrm{AOB} = 100^\circ$ and $\angle\mathrm{AOC} = 90^\circ$ then $\angle\mathrm{BAC} = ?$

(A) 85° (B) 80° (C) 95° (D) 75°



Ans. :

a. 85°

Solution:

$\angle BOA + \angle AOC + \angle BOC = 360^\circ$ [Angles around a point are 360°]

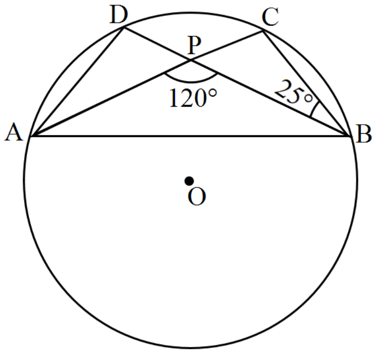
$\Rightarrow\ 100^\circ + 90^\circ + \angle BOC = 360^\circ$

$\Rightarrow\ \angle BOC = 170^\circ$

Now,

$\angle BAC = \frac{1}{2}(\angle BOC) = \frac{1}{2}(170^\circ) = 85^\circ$

4. O is the centre of the given circle. If $\angle APB = 120^\circ$ and $\angle DBC = 25^\circ$, then the measure of $\angle ADB$ is equal to:

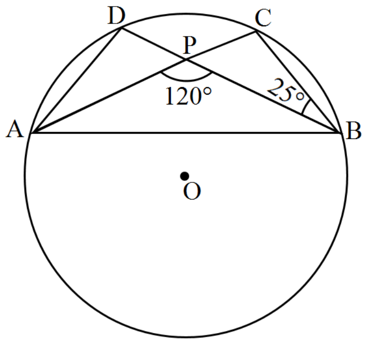


(A) 60º (B) 120º (C) 95º (D) 100º

Ans. :

c. 95º

Solution:



Now, $\angle APB + \angle CPB = 180^\circ$ (Linear Pair)

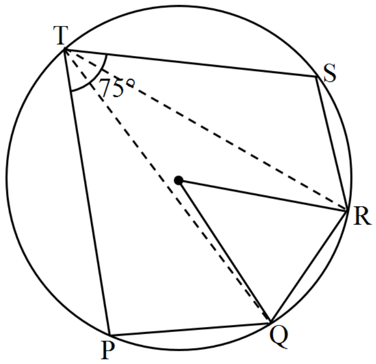
$120^\circ + \angle CPB = 180^\circ$

$\angle CPB = 60^\circ$

Now from angle sum property, we can calculate the values of $\angle CPB$ and we find that $\angle CPB = 95^\circ$

Since, $\angle PCB = \angle ADB = 95^\circ$

5. In the given figure PQ = QR = RS and $\angle PTS = 75^\circ$ then the measure of $\angle QOR$ is:

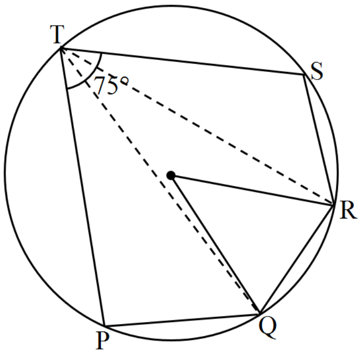


(A) 75º (B) 25º (C) 50º (D) 20º

Ans. :

c. 50º

Solution:



$\angle \mathrm{PTS}=75^{\circ}$

Now $\angle \mathrm{PTQ}=\angle \mathrm{QTR}=\angle \mathrm{RTS}$ (Equal chords would make equal angles at centre and thus equal angles at the circumference)

$\angle \mathrm{QTR}=\frac{75^{\circ}}{3}=25^{\circ}$

So, $\angle \mathrm{QOR}=25^{\circ} \times 2=50^{\circ}$

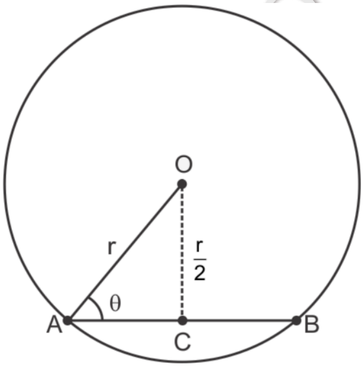
6. If O is the centre of a circle of radius r and AB is a chord of the circle at a distance $\frac{r}{2}$ from O, then $\angle \mathrm{BAO}=$

(A) 60° (B) 45° (C) 30° (D) 15°

Ans. :

c. 30°

Solution:



Let $\angle \mathrm{BAO}=\theta$

Consider $\triangle \mathrm{OAC}$,

$\sin\theta = \frac{\mathrm{OC}}{\mathrm{OA}} = \frac{\frac{\mathrm{r}}{2}}{\mathrm{r}}$
$= \frac{1}{2} = \sin 30^\circ$
$\Rightarrow \theta = 30^\circ$

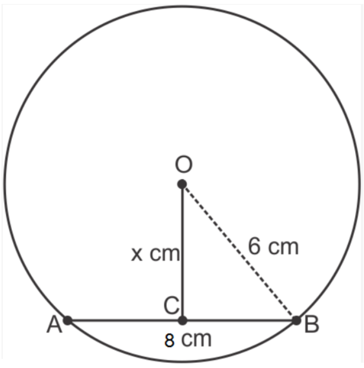
7. The radius of a circle is 6cm. The perpendicular distance from the centre of the circle to the chord which is 8cm in length, is:

(A) $\sqrt{5}\mathrm{cm.}$ (B) $2\sqrt{5}\mathrm{cm.}$ (C) $2\sqrt{7}\mathrm{cm.}$ (D) $\sqrt{7}\mathrm{cm.}$

Ans. :

b. $2\sqrt{5}\mathrm{cm.}$

Solution:



AB = 8cm

⇒ AC = BC = 4cm

Consider $\triangle\mathrm{OCB}$, where BC = 8cm,

OB = 6cm

Now, $(\mathrm{OC})^2 + (\mathrm{BC})^2 = (\mathrm{OB})^2$

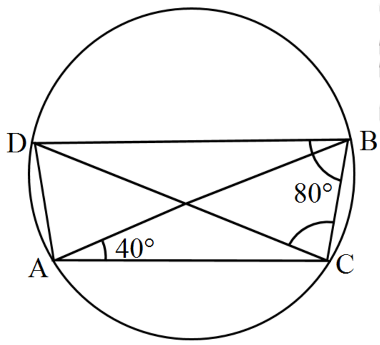
⇒ $(\mathrm{OC})^2 + 4^2 = 6^2$

⇒ $(\mathrm{OC})^2 + 16 = 36$

⇒ $(\mathrm{OC})^2 = 20$

$\Rightarrow \mathrm{OC} = \sqrt{20} = 2\sqrt{5}$

8. In the given figure, AB and CD are two intersecting chords of a circle. If $\angle\mathrm{CAB} = 40^\circ$ and $\angle\mathrm{BCD} = 80^\circ$, then $\angle\mathrm{CBD} = ?$



(A) 80º (B) 70º (C) 60º (D) 50º

Ans. :

c. 60º

Solution:

We have:

$\angle CDB = \angle CAB = 40^\circ$ (Angles in the same segment of a circle)

In $\triangle CBD$, we have:

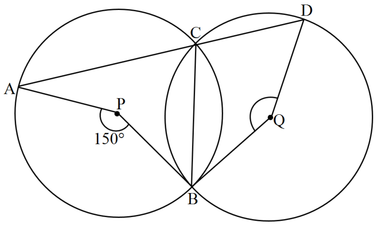
$\angle CDB + \angle BCD + \angle CBD = 180^\circ$ (Angle sum property of a triangle)

$\Rightarrow 40^\circ + 80^\circ + \angle CBD = 180^\circ$

$\Rightarrow \angle CBD = (180^\circ - 120^\circ) = 60^\circ$

$\Rightarrow \angle CBD = 60^\circ$

9. In the given figure, P and Q are centers of two circles intersecting at B and C. ACD is a straight line. Then, the measure of $\angle BQD$ is:

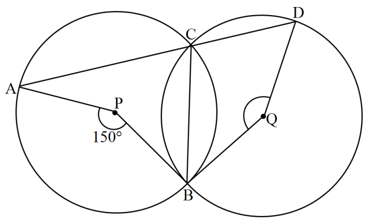


(A) 115º (B) 150º (C) 105º (D) 130º

Ans. :

b. 150º

Solution:



$\angle APB = 150^\circ$, so, $\angle ACB = 75^\circ$ {Angle subtended by an arc at centre is twice the angle subtended at any point on circumference}

Now, ACD is straight line, so, $\angle ACB + \angle DCB = 180^\circ$

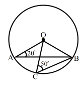
$\angle DCB = 180 - 75 = 105^\circ$

Now, angle subtended by arc BD on centre is twice of $\angle DCB = 2 \times 105 = 210^\circ$

Now, $\angle BQD = 360^\circ - 210^\circ = 150^\circ$

10. In the given figure, O is the centre of a circle in which $\angle OAB = 20^\circ$ and $\angle OCB = 50^\circ$. Then, $\angle AOC = ?$

(A) 50° (B) 70° (C) 20° (D) 60°



Ans. :

d. 60°

Solution:

OA = OC [Radii of the same circle]

$\Rightarrow \angle OBA = \angle OAB = 20^\circ$

In $\triangle OAB$,

$\angle OBA + \angle OAB + \angle AOB = 180^\circ$ [Angle sum property]

$\Rightarrow\ 20^\circ + 20^\circ + \angle\mathrm{AOB} = 180^\circ$

$\Rightarrow\ \angle\mathrm{AOB} = 140^\circ$

Now,

OB = OC [Radii of the same circle]

$\Rightarrow\ \angle\mathrm{OBC} = \angle\mathrm{OCB} = 50^\circ$

In $\triangle\mathrm{OCB},$

$\angle\mathrm{OBC} + \angle\mathrm{OCB} + \angle\mathrm{COB} = 180^\circ$ [Angle sum property]

$\Rightarrow\ 50^\circ + 50^\circ + \angle\mathrm{COB} = 180^\circ$

$\Rightarrow\ \angle\mathrm{COB} = 80^\circ$

So,

$\angle\mathrm{AOB} = \angle\mathrm{AOC} + \angle\mathrm{COB}$

$\Rightarrow\ \angle\mathrm{AOC} = \angle\mathrm{AOB} - \angle\mathrm{COB}$

$\Rightarrow\ \angle\mathrm{AOC} = 140^\circ - 80^\circ$

$\Rightarrow\ \angle\mathrm{AOC} = 60^\circ$

11. AB and CD are two parallel chords of a circle with centre O such that AB = 6cm and CD = 12cm. The chords are on the same side of the centre and the distance between them is 3cm. The radius of the circle, is:

(A) $7\mathrm{cm}$ (B) $6\mathrm{cm}$ (C) $3\sqrt{5}\mathrm{cm}$ (D) $5\sqrt{2}\mathrm{cm}$

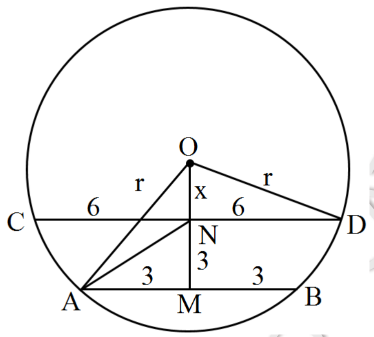
Ans. :

c. $3\sqrt{5}\mathrm{cm}$

Solution:

Let the distance between the center and the chord CD be x cm and the radius of the circle is r cm.

We have to find the radius of the following circle:



In right angled triangle, OND,

$x^2 + 36 = r^2$ ....(i)

Now, in right angled triangle AOM,

$r^2 = 9 + (x + 3)^2$ ....(ii)

From (i) and (ii), we have

$\mathrm{r}^2 = 9 + ((\sqrt{\mathrm{r}})^2 - 36 + 3)^2$

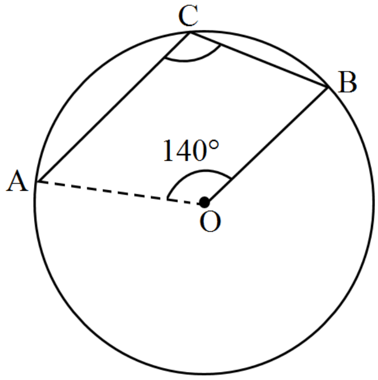
$\Rightarrow \mathrm{r}^2 = 9 + \mathrm{r}^2 - 36 + 9 + 6\sqrt{\mathrm{r}^2 - 36}$

$\Rightarrow 3 = \sqrt{\mathrm{r}^2 - 36}$

$\Rightarrow 9 = \mathrm{r}^2 - 36$ [squaring both the sides]

$$\Rightarrow r^2 = 45 \Rightarrow r = 3\sqrt{5}\text{cm}$$

12. In the given figure, O is the centre of a circle and $\angle AOB = 140^\circ$. Then, $\angle ACB = ?$



(A) 110º (B) 70º (C) 80º (D) 40º

Ans. :

a. 110º

Solution:

Let, D on any point on circumference and join AD and BD,

Now, $\angle ADB = \frac{\angle AOB}{2}$

$\angle ADB = \frac{140}{2} = 70^\circ$

Now, in cyclic quadrilateral ADBC

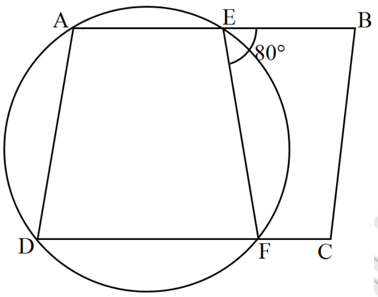
$\Rightarrow \angle ADB + \angle ACB = 180^\circ$

$\Rightarrow \angle ACB = 180^\circ - \angle ADB$

$\Rightarrow \angle ACB = 180^\circ - 70^\circ$

$\Rightarrow \angle ACB = 110^\circ$

13. ABCD is a parallelogram. A circle passes through A and D and cuts AB at E and DC at F. If $\angle BEF = 80^\circ$, then $\angle ABC$ is equal to:

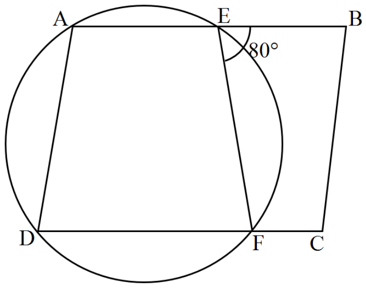


(A) 75º (B) 80º (C) 100º (D) 120º

Ans. :

b. 80º

Solution:



$\angle AEF + 80^\circ = 180^\circ$ (Linear Pair)

$\angle AEF = 100^\circ$

$\angle ADF + \angle AEF = 180^\circ$ (Opposite angles of a cyclic quadrilateral)

$\angle ADF = 180^\circ - 100^\circ = 80^\circ$

$\angle ADF = \angle ABC = 80^\circ$ (Opposite angles of a parallelogram)

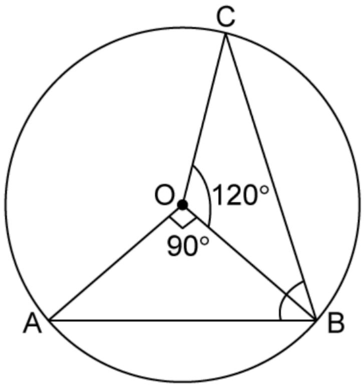
14. If A , B, C are three points on a circle with centre O such that $\angle AOB = 90^\circ$ and $\angle BOC = 120^\circ$, then $\angle ABC =$

(A) 60° (B) 75° (C) 90° (D) 135°

Ans. :

b. 75°

Solution:



$\angle AOC = \angle AOB + \angle BOC$

$= 90^\circ + 120^\circ = 210^\circ$

$\angle COA = 360^\circ - 210^\circ = 150^\circ$

If arc $\widehat{COA}$ makes 150° at centre, then it will make half angle of the centre at circumference.

$\Rightarrow \angle CBA$ or $\angle ABC = \frac{150^\circ}{2} = 75^\circ$

15. Number of circles that can be drawn through three non-collinear points is:

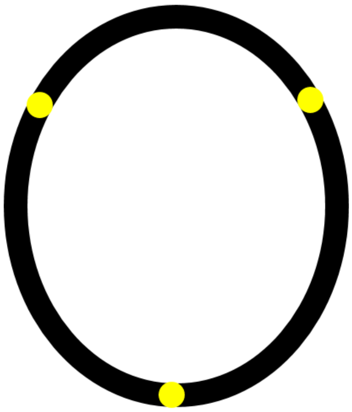
(A) 2 (B) 1 (C) 0 (D) 3

Ans. :

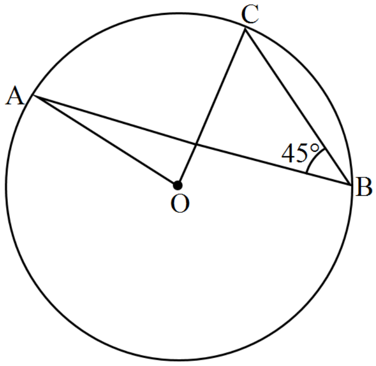
b. 1

Solution:

Only 1 circle can be drawn from three non-collinear points.



16. In the given figure, if $\angle \mathrm{ABC} = 45^{\circ}$, then $\angle \mathrm{AOC} =$



(A) 75º (B) 45º (C) 90º (D) 60º

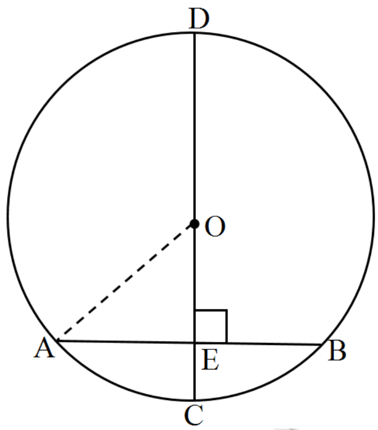
Ans. :

c. 90º

Solution:

The angle made by an arc at the centre is double the angle made by it on any other point on the circumfrence.

17. In the given figure, CD is the diameter of a circle with centre O and CD is perpendicular to chord AB. If AB = 12cm and CE = 3cm, then radius of the circles is:



(A) 9cm (B) 6cm (C) 8cm (D) 7.5cm

Ans. :

d. 7.5cm

Solution:

Let OA = OC = r cm.

Then OE = (r - 3)cm and $\mathrm{AE} = \frac{1}{2}\mathrm{AB} = 6\mathrm{cm}$

Now, in right $\triangle OAE$, we have:

$OA^2 = OE^2 + AE^2$ [Using paythagoras theorem]

$\Rightarrow (r)^2 = (r - 3)^2 + 6^2$

$\Rightarrow r^2 = r^2 + 9 - 6r + 36$

$\Rightarrow 6r = 45$

$\Rightarrow r = \frac{45}{6} = 7.5cm$

Hence, the required radius of the circle is 7.5cm.

18. An angle in the semicircle is:

(A) 360º (B) None of these. (C) 180º (D) 90º

Ans. :

d. 90º

Solution:

The angle in a semicircle is always 90º.

19. Two circle are congruent if they have equal.

(A) Radius (B) Diameter (C) Secant (D) Chord

Ans. :

a. Radius

Solution:

Equal radius would generate two same circles that are exact copy of each other, hence making them congruent.

20. A circle is drawn. It divides the plane into:

(A) No Parts (B) 4 Parts (C) 5 Parts (D) 3 Parts

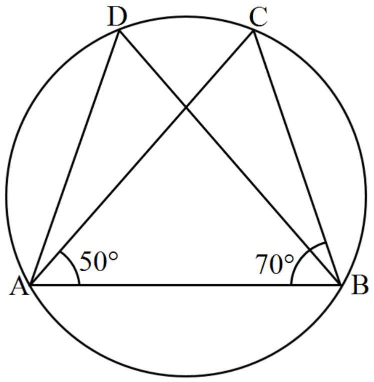
Ans. :

d. 3 Parts

Solution:

A circle divides the plane into 3 parts namely, the points outside the circle, the points inside the circle and the points on the circle.

21. In the given figure, if $\angle CAB = 50^\circ$ and $\angle ABC = 70^\circ$, then $\angle ADB$ is equal to:



(A) 60º (B) 80º (C) 70º (D) 50º

Ans. :

a. 60º

Solution:

In triangle ABC, $\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow \angle C = 60^\circ$

$\angle ACB = \angle ADB = 60^\circ$ (Angle made by the same chord are equal)

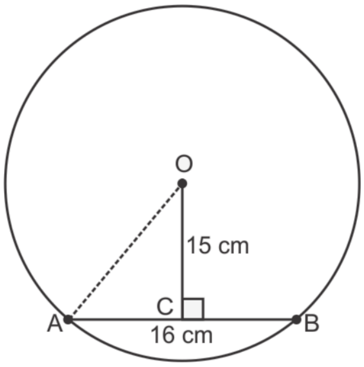
22. If the length of a chord of a circle is 16cm and is at a distance of 15cm from the centre of the circle, then the radius of the circle is:

(A) 15cm. (B) 16cm. (C) 17cm. (D) 34cm.

Ans. :

c. 17cm.

Solution:



AB = 16cm

OC = 15cm

C is the mid-point of AB.

$AC = BC = \frac{16}{2} = 8\text{cm}$

Consider $\triangle OCA$,

$OC = 15\text{cm},\ AC = 8\text{cm}$

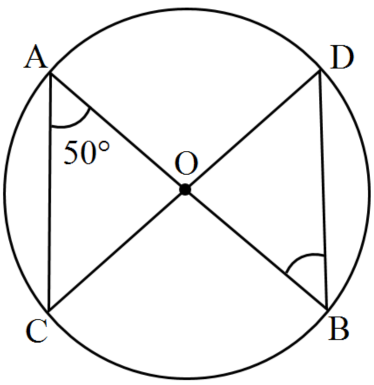
$\Rightarrow OA = \sqrt{(15)^2 + (8)^2}$

$= \sqrt{225 - 64}$

$= \sqrt{289}$

$\Rightarrow OA = 17\text{cm}$

23. In the given figure, O is the centre of a circle. If $\angle OAC = 50^\circ$, then $\angle ODB = ?$



(A) 40º (B) 50º (C) 75º (D) 60º

Ans. :

b. 50º

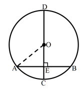
Solution:

$\angle\mathrm{ODB} = \angle\mathrm{OAC} = 50^\circ$ (Angles in the same segment of a circle)

$\Rightarrow \angle\mathrm{ODB} = 50^\circ$

24. In the given figure, CD is the diameter of a circle with centre O and CD is perpendicular to chord AB. If AB = 12cm and CE = 3cm, then radius of the circles is:

(A) 6cm (B) 9cm (C) 7.5cm (D) 8cm



Ans. :

c. 7.5cm

Solution:

OA = OC

⇒ OA = OE + CE

⇒ OA = OE + 3

⇒ OE = OA - 3 ...(i)

$\mathrm{AE} = \frac{1}{2}\mathrm{AB}$ [Perpendicular drawn from the centre of a circle to the chord bisect the chord]

$= \frac{1}{2}(12) = 6\mathrm{cm}$

In right $\triangle\mathrm{OEA},$

$OA^2 = OE^2 + AE^2$

⇒ $OA^2 = (OA - 3)^2 + AE^2$ [From (i)]

⇒ $OA^2 = OA^2 - 6OA + 9 + AE^2$

⇒ $6OA = 9 + 6^2$

⇒ 6OA = 9 + 36

$\Rightarrow\ \mathrm{OA} = \frac{45}{6} = 7.5\mathrm{cm}$

So, the radius of the circle is 7.5cm.

25. Circle having same centre are said to be:

(A) Secant (B) Concentric (C) Chord (D) Circle

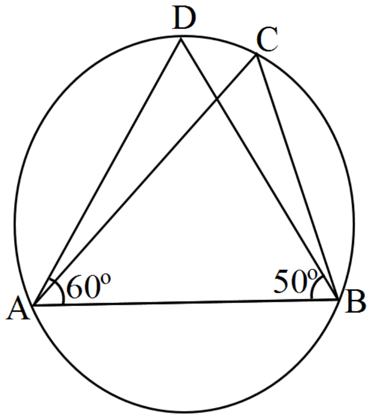
Ans. :

b. Concentric

Solution:

Concentric circles are those circle that is drawn with the same point as a centre but different radii.

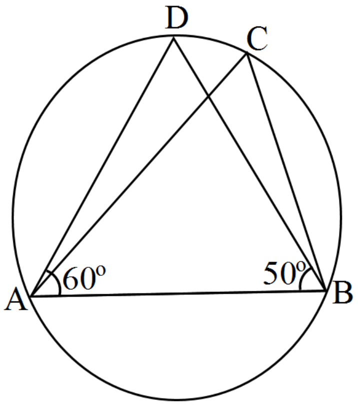
26. In the figure, if $\angle\mathrm{DAB} = 60^\circ, \angle\mathrm{ABD} = 50^\circ$ then $\angle\mathrm{ACB}$ is equal to:



(A) 80º (B) 60º (C) 50º (D) 70º

Ans. :
d. 70º

Solution:



In, $\triangle ABD$

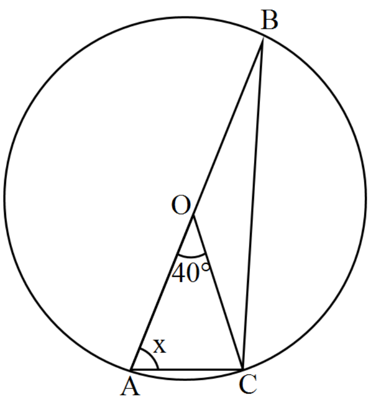
$\angle D = 180^\circ - \angle A - \angle B$

$= 180^\circ - 110^\circ = 70^\circ$

Since angles made by same chord at any point of circumference are equal so,

$\angle ACB = \angle ADB = 70^\circ$

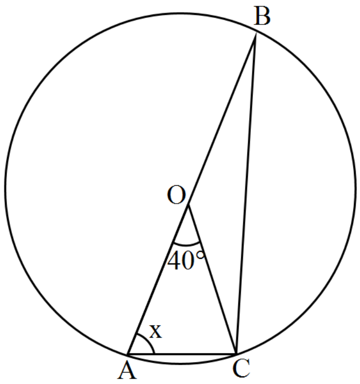
27. In a figure, O is the centre of the circle with AB as diameter. If $\angle AOC = 40^\circ$, the value of x is equal to:



(A) 60º (B) 70º (C) 50º (D) 80º

Ans. :
b. 70º

Solution:



OA = OC (radii)

So, $\angle OAC = \angle OCA = x$

Again, in $\triangle OAC$

$\angle OAC + \angle OCA + \angle AOC = 180^\circ$

$x + x + \angle AOC = 180^\circ$

$x + x + 40^\circ = 180^\circ$

$2x = 140^\circ$

$x = 70^\circ$

28. Write the correct answer in the following:
ABCD is a cyclic quadrilateral such that AB is a diameter of the circle circumscribing it and $\angle ADC = 140^\circ$, then $\angle BAC$ is equal to:

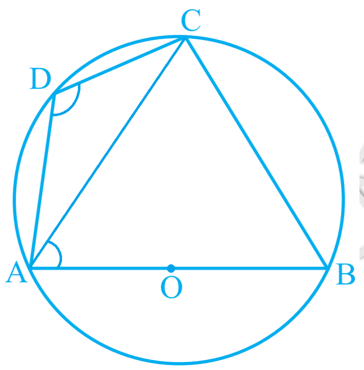
(A) 80°. (B) 50°. (C) 40°. (D) 30°.

Ans. :

b. 50°.

Solution:

Given, ABCD is a cyclic quadrilateral and $\angle ADC = 140^\circ$.



We know that, sum of the opposite angles in a cyclic quadrilateral is 180°.

$\angle ADC + \angle ABC = 180^\circ$

$\Rightarrow\ 140^\circ + \angle ABC = 180^\circ$

$\Rightarrow\ \angle ABC = 180^\circ - 140^\circ$

$\therefore \angle ABC = 40^\circ$

Since, $\angle ACB$ is an angle in a semi-circle.

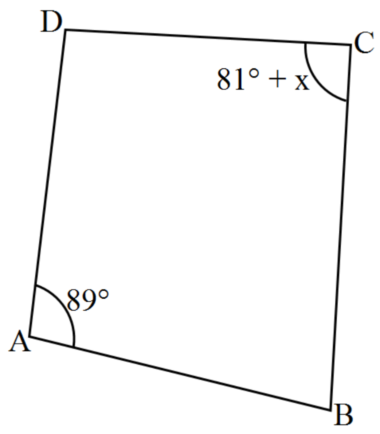
$\therefore \angle ACB = 90^\circ$

In $\triangle ABC,\ \ \angle BAC + \angle ACB + \angle ABC = 180^\circ$ [by angle sum property of a triangle]

$\Rightarrow \angle BAC + 90^\circ + 40^\circ = 180^\circ$

$\Rightarrow \angle BAC = 180^\circ - 130^\circ = 50^\circ$

29. For what value of x in the figure, points A, B, C and D are concyclic?

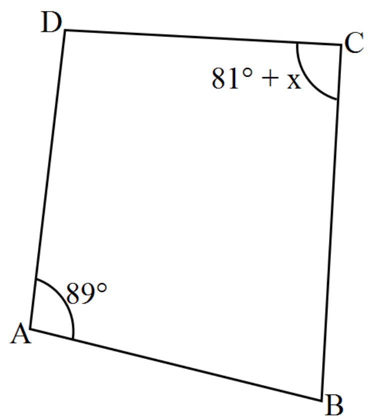


(A) 9º (B) 10º (C) 11º (D) 12º

Ans. :

b. 10º

Solution:



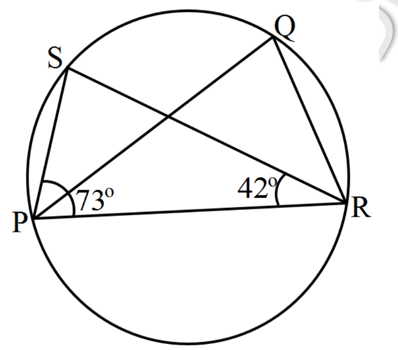
If the quadrilateral ABCD is concyclic, then,

$\angle A + \angle C = 180^\circ$

80º + 81º + x = 180º

x = 10º

30. In the figure, if $\angle SPR = 73^\circ, \angle SRP = 42^\circ$ then $\angle PQR$ is equal to:

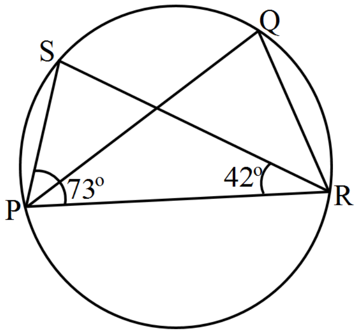


(A) 74º (B) 76º (C) 70º (D) 65º

Ans. :

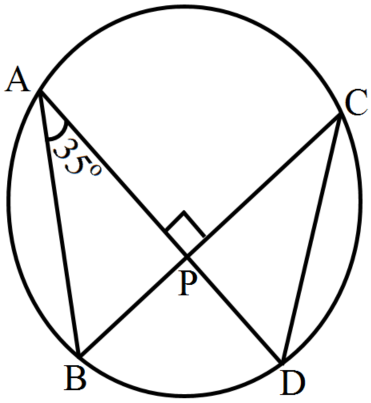
d. 65º

Solution:



$\angle PQR = \angle PSR = 180^\circ - 73^\circ - 42^\circ = 65^\circ$

31. Chords AD and BC intersect each other at right angles at point P. $\angle DAB = 35^\circ$, then $\angle ADC$ is equal to:



(A) 35º (B) 45º (C) 65º (D) 55º

Ans. :

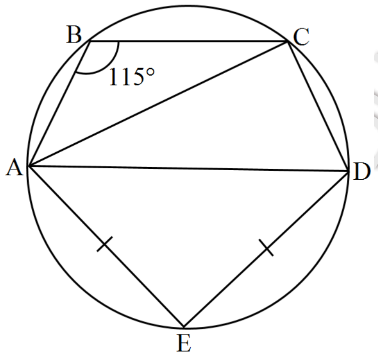
d. 55º

Solution:

From triangle APB, $\angle ABP = 180^\circ - 90^\circ - 35^\circ = 55^\circ$

Thus, $\angle ADC = 55^\circ$ $(\angle ABC = \angle ADC)$

32. In the given figure, AD is the diameter of the circle and AE = DE. If, $\angle ABC = 115^\circ$, then the measure of $\angle CAE$ is:

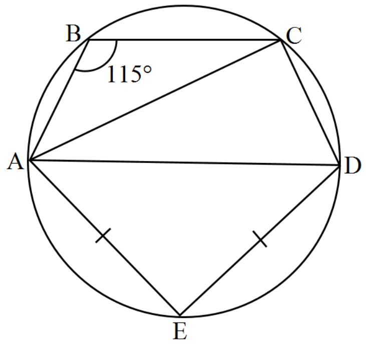


(A) 70º (B) 60º (C) 80º (D) 90º

Ans. :

a. 70º

Solution:



Since, AE = DE,

therefore, $\angle DAE = \angle ADE = 45^\circ$ (In $\triangle AED, \angle E$ And other two angles are equal.)

Now, BADC is a cyclic quadrilateral,

$\Rightarrow \angle B + \angle D = 180^\circ$

$\Rightarrow 115^\circ + \angle D = 180^\circ$

$\Rightarrow \angle CDA = \angle D = 65^\circ$

Also, $\angle ACD = 90^\circ$ (Angle in a semicircle)

So, we have:-

In $\triangle ACD,$

$\Rightarrow \angle CAD + \angle ACD + \angle CDA = 180^\circ$

$\Rightarrow \angle CAD + 90^\circ + 65^\circ = 180^\circ$

$\Rightarrow \angle CAD = 25^\circ$

Finally,

$\angle CAE = \angle CAD + \angle DAE = 25^\circ + 45^\circ = 70^\circ$

33. In a circle with centre O, AB and CD are two diameters perpendicular to each other. The length of chord AC, is:

(A) $\frac{1}{\sqrt{2}\,AB}$ (B) $\sqrt{2}$ (C) $\frac{1}{2AB}$ (D) $2AB$

Ans. :

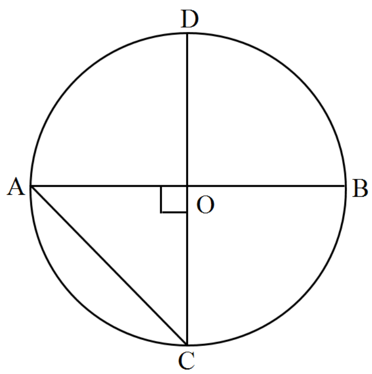
a. $\frac{1}{\sqrt{2}\,AB}$

Solution:

We are given a circle with centre at O and two perpendicular diameters AB and CD.

We need to find the length of AC

We have the following corresponding figure:



Since, AB = CD (Diameter of the same circle)

Also, $\angle AOC = 90^\circ$

And, $AO = \frac{AB}{2}$

Here, AO = OC (radius)

In $\triangle AOC$ is right angled triangle,

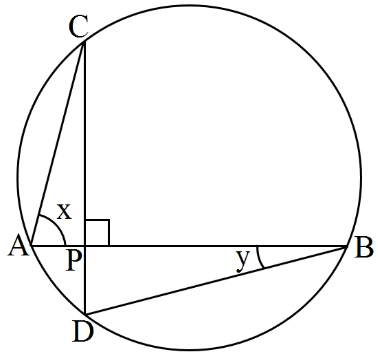
$AC^2 = AO^2 + OC^2 = AO^2 + AO^2$

$= \left(\frac{AB}{2}\right)^2 + \left(\frac{AB}{2}\right)^2$

$AC2 = \frac{AB^2}{2}$

$AC = \frac{AB}{\sqrt{2}}$

34. In the given figure, if chords AB and CD of the circle intersect each other at right angles, then, x + y =

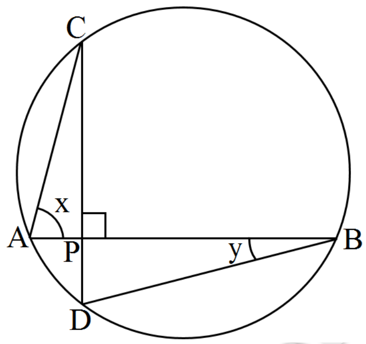


(A) 90º (B) 60º (C) 75º (D) 45º

Ans. :

a. 90º

Solution:



$y = \angle ACP$ (Angles of same arc)

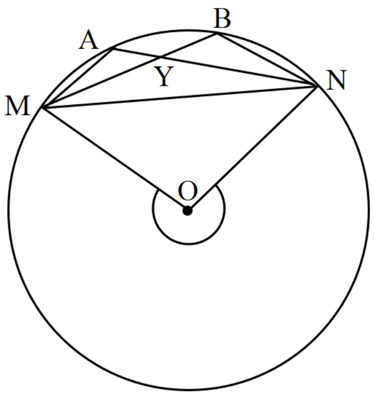
$\angle APC = 180^\circ - 90^\circ = 90^\circ$ ($\angle APC, \angle CPB$ are linear pair)

Thus from triangle APC

$x + y + \angle APC = 180^\circ$

Hence, x + y = 90º

35. In the given figure, M, A, B and N are points on a circle having centre O. AN and MB cut at Y. If $\angle NYB = 50^\circ$ and $\angle YNB = 20^\circ$, then reflex $\angle MON$ is equal to:

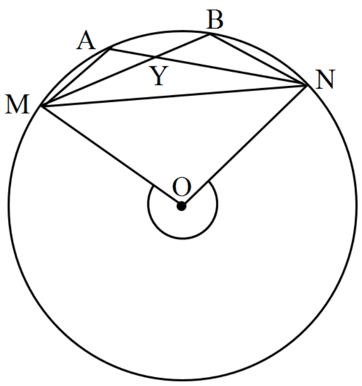


(A) 260º (B) 200º (C) 220º (D) 240º

Ans. :

c. 220º

Solution:



In triangle NYB,

$\angle \mathrm{N} + \angle \mathrm{Y} + \angle \mathrm{B} = 180^\circ$

$\Rightarrow \angle \mathrm{B} = 180^\circ - 50^\circ - 20^\circ = 110^\circ$

Complete the cyclic quadrilateral, MBNX, where X being any point on the circumference in the major segment, we have:-

$\angle \mathrm{MXN} = 80^\circ - 110^\circ = 70^\circ$

So, minor $\angle \mathrm{MON} = 70^\circ \times 2 = 140^\circ$

Hence, reflex $\angle \mathrm{MON} = 360^\circ - 140^\circ = 220^\circ$

36. The relation between diameter and radius of a circle is:

(A) $\mathrm{d} = 2\pi\mathrm{r}$ (B) d = 2r (C) r = 2d (D) d = r

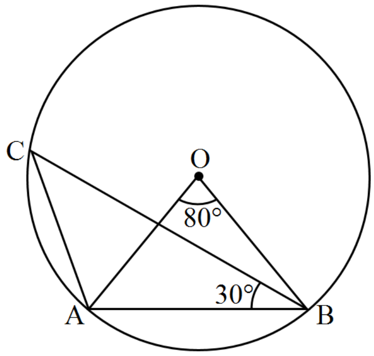
Ans. :

b. d = 2r

Solution:

Radius is half the length of the diameter, thus diameter is twice the length of the radius.

37. In the given figure, if $\angle \mathrm{AOB} = 80^\circ$ and $\angle \mathrm{ABC} = 30^\circ$, then $\angle \mathrm{CAO}$ is equal to:

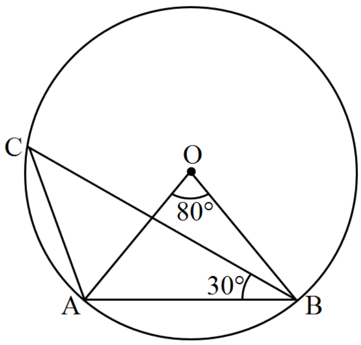


(A) 60º (B) 40º (C) 80º (D) 30º

Ans. :

a. 60º

Solution:



In triangle AOB, OA = OB (Radii)

$\Rightarrow \angle \mathrm{OAB} = \angle \mathrm{OBA}$

Now,

$\Rightarrow \angle \mathrm{OAB} + \angle \mathrm{ABO} + \angle \mathrm{AOB} = 180^\circ$

$\Rightarrow 2\angle \mathrm{OAB} = 80^\circ = 180^\circ$

$\Rightarrow \angle \mathrm{OAB} = 50^\circ$

Now, $\angle \mathrm{CAB} = \frac{1}{2}\angle \mathrm{AOB} = \frac{80^\circ}{2} = 40^\circ$

Again, in triangle ABC,

$\angle \mathrm{CAB} + \angle \mathrm{ABC} + \angle \mathrm{BCA} = 180^\circ$

$\angle \mathrm{CAB} + 30^\circ + 40^\circ = 180^\circ$

$\angle \mathrm{CAB} = 110^\circ$

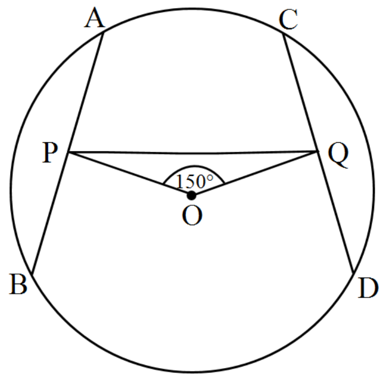
Thus,

$\angle \mathrm{CAO} = \angle \mathrm{CAB} - \angle \mathrm{OAB}$

$= 110^\circ - 50^\circ = 60^\circ$

Hence, proved.

38. In Fig., AB and CD are two equal chords of a circle with centre O. OP and OQ are perpendiculars on chords AB and CD, respectively. If $\angle \mathrm{POQ} = 150^\circ$, then $\angle \mathrm{APQ}$ is equal to:

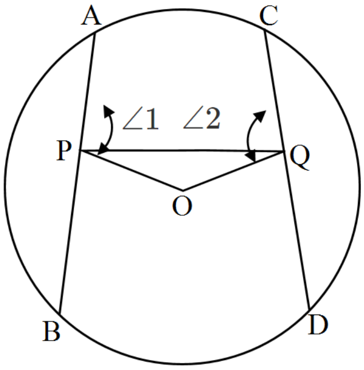


(A) 60º (B) 30º (C) 15º (D) 75º

Ans. :

d. 75º

Solution:



As AB = CD

So, OP = OQ (equal chords are equidistant from the centre)

$\angle 1=\angle 2$ (angles opposite to equal sides are equal)

$\angle 1+\angle 2+\angle \mathrm{POQ}=180^{\circ}$

$\angle 1+\angle 1+150^{\circ}=180^{\circ}$

$\therefore \angle 1=15^{\circ}$

Since APB is a line segement

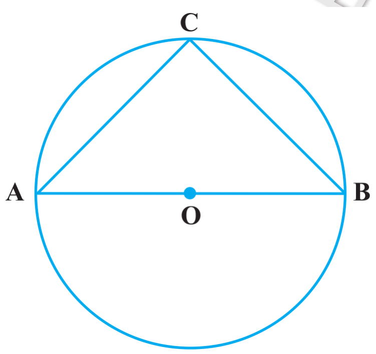
$\therefore \angle \mathrm{BPO}+\angle 1+\angle \mathrm{APQ}=180^{\circ}$

$90^{\circ}+15^{\circ}+\angle \mathrm{APQ}=180^{\circ}$

$\therefore \angle \mathrm{APQ}=75^{\circ}$

39. Write the correct answer in the following:

In Fig. if AOB is a diameter of the circle and AC = BC, then $\angle \mathrm{CAB}$ is equal to:



(A) 30°. (B) 60°. (C) 90°. (D) 45°.

Ans. :

d. 45°.

Solution:

As AOB is a diameter of the circle,

$\angle C = 90^\circ$

[∵ Angles in a semi-circle is 90°]

Now, AC = BC

$\angle A = \angle B$

[∵ Angles opposite to equal sides of triangle are equal]

Using angle sum property of a triangle, we have

$\angle A + \angle B + \angle C = 180^\circ \Rightarrow 2\angle A + 90^\circ = 180^\circ$

$\Rightarrow 2\angle A = 90^\circ \Rightarrow \angle A = 90^\circ \div 2 = 45^\circ$

Hence, (d) is the correct answer.

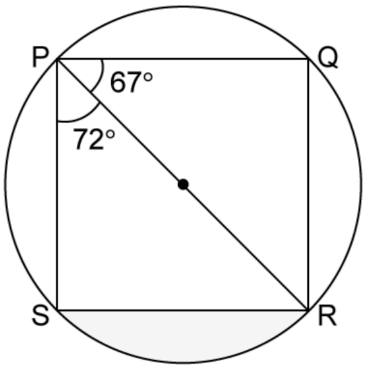
40. PQRS is a cyclic quadrilateral such that PR is a diameter of the circle. If $\angle QPR = 67^\circ$ and $\angle SPR = 72^\circ$, then $\angle QRS =$

(A) 41° (B) 23° (C) 67° (D) 18°

Ans. :

a. 41°

Solution:



In a cyclic quadrilateral, Opposite angles are supplementary.

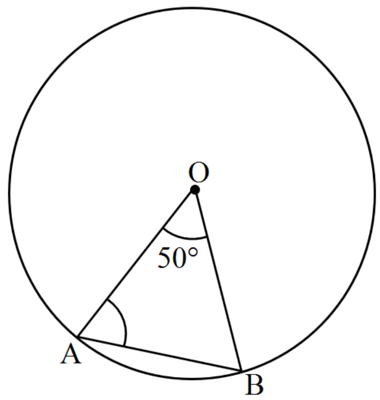
$\Rightarrow \angle P + \angle R = 180^\circ$

Now, $\angle P = 67^\circ + 72^\circ = 139^\circ$

Thus, $\angle R = 180^\circ - 139^\circ = 41^\circ$

i.e. $\angle R = \angle QRS = 41^\circ$

41. In the given figure, O is the centre of a circle. Then, $\angle OAB =$



(A) 50º (B) 55º (C) 60º (D) 65º

Ans. :

d. 65º

Solution:

OA = OB (Radii of a circle)

Let $\angle \mathrm{OAB} = \angle \mathrm{OBA} = \mathrm{x}^{\circ}$

In $\triangle \mathrm{OAB}$, we have:

xº + xº + 50º = 180º (Angle sum property of a triangle)

⇒ 2xº = (180º - 50º) = 130º

$\Rightarrow \mathrm{x}^{\circ} = (\frac{130}{2})^{\circ} = 65^{\circ}$

Hence, $\angle \mathrm{OAB} = \mathrm{x}^{\circ} = 65^{\circ}$

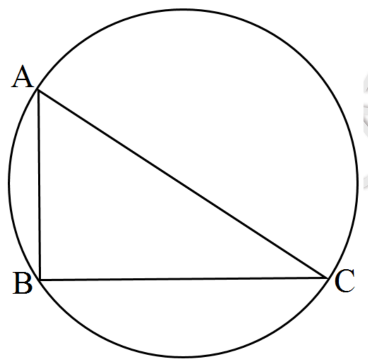
42. If AB = 12cm, BC = 16 and AB is perpendicular to BC, then the radius of the circle passing through the points A, B and C is:.

(A) 12cm (B) 6cm (C) 8cm (D) 10cm

Ans. :

d. 10cm

Solution:



Since AB is perpendicular to BC, therefore ABC is a right-angled triangle right angled at B. As clear from the figure, AC would act as the diameter

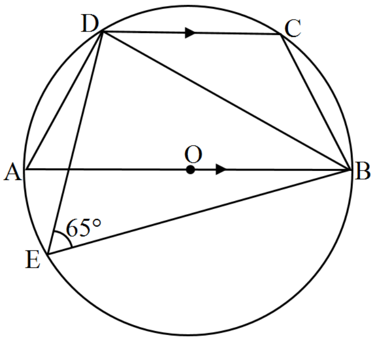
$AB^2 + BC^2 = AC^2$

$12^2 + 16^2 = AC^2$

AC = 20

Since AC is diameter so radius = 10cm.

43. AOB is the diameter of the circle. ABCD is a cyclic trapezium in which AB || DC. If $\angle \mathrm{BED} = 65^{\circ}$, then $\angle \mathrm{BDC}$ is equal to:

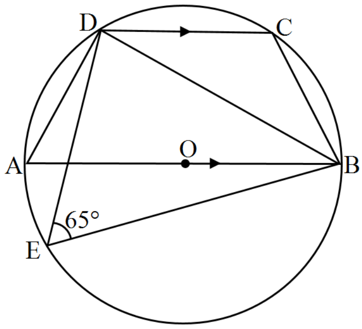


(A) 40º (B) 25º (C) 65º (D) 75º

Ans. :

b. 25º

Solution:



AB is diameter and ABCD is semi-circle, so $\angle \mathrm{ADB} = 90^\circ$

Also, $\angle \mathrm{BED} = \angle \mathrm{BAD} = 65^\circ$ {Angles in same segment}

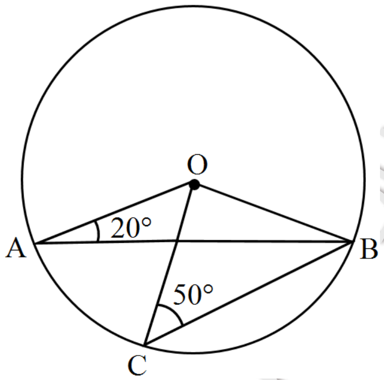
so, in $\triangle \mathrm{ABD}, \angle \mathrm{BAD} + \angle \mathrm{ABD} + \angle \mathrm{ADB} = 180^\circ$

$\angle \mathrm{ADB} = 180 - 155 = 25^\circ$

Now, since, AB || CD

So, $\angle \mathrm{ADB} = \angle \mathrm{BDC} = 25^\circ$ {alternate interior angles}

44. In the given figure, O is the centre of a circle in which $\angle \mathrm{OAB} = 20^\circ$ and $\angle \mathrm{OCB} = 50^\circ$. Then, $\angle \mathrm{AOC} = ?$



(A) 20º (B) 60º (C) 70º (D) 50º

Ans. :

b. 60º

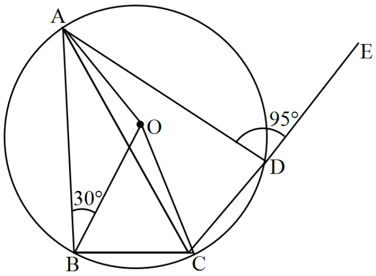
Solution:

$\mathrm{OA} = \mathrm{OB} \Rightarrow \angle \mathrm{OBA} = \angle \mathrm{OAB} = 20^\circ.$

In $\triangle \mathrm{OAB},$

$\angle \mathrm{OAB}+\angle \mathrm{OBA}+\angle \mathrm{AOB}=180^{\circ}$
$\Rightarrow 20^{\circ}+20^{\circ}+\angle \mathrm{AOB}=180^{\circ}$
$\Rightarrow \angle \mathrm{AOB}=140^{\circ}$.
$\mathrm{OB}=\mathrm{OC} \Rightarrow \angle \mathrm{OBC}=\angle \mathrm{OCB}=50^{\circ}$.
In $\triangle \mathrm{OCB}$,
$\angle \mathrm{OCB}+\angle \mathrm{OBC}+\angle \mathrm{COB}=180^{\circ}$
$\Rightarrow 50^{\circ}+50^{\circ}+\angle \mathrm{COB}=180^{\circ}$
$\Rightarrow \angle \mathrm{COB}=80^{\circ}$.
$\angle \mathrm{AOB}=140^{\circ} \Rightarrow \angle \mathrm{AOC}+\angle \mathrm{COB}=140^{\circ}$
$\Rightarrow \angle \mathrm{AOC}+80^{\circ}=140^{\circ}$
$\Rightarrow \angle \mathrm{AOC}=140^{\circ}-80^{\circ}$
$\Rightarrow \angle \mathrm{AOC}=60^{\circ}$.

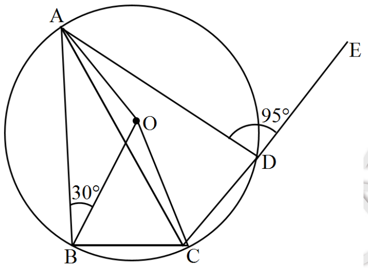
45. In the given figure, ABCD is a quadrilateral inscribed in circle with centre O. CD is produced to E. If $\angle \mathrm{ADE}=95^{\circ}$ and $\angle \mathrm{OBA}=30^{\circ}$, then $\angle \mathrm{OAC}$ is equal to:



(A) 10º (B) 20º (C) 15º (D) 5º

Ans. :
d. 5º

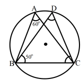
Solution:



Here, $\angle \mathrm{ADC}$ and $\angle \mathrm{ADE}$ are supplementary to each other,
So, $\angle \mathrm{ADC}=180-95=85^{\circ}$
Also, ABCD is also a cyclic quadrilateral so, $\angle \mathrm{ADC}$ and $\angle \mathrm{ABC}$ are supplementary
So, $\angle \mathrm{ABC}=\angle \mathrm{ADC}=180^{\circ}$
$\angle \mathrm{ABC}=180-95=85^{\circ}$
angle subtended at centre is double the angle suntended at circumference.
So, $\angle \mathrm{AOC}=170^{\circ}$
Now, in triangle AOC, OA = OC(Radii) so, $\angle \mathrm{OAC}=\mathrm{OCA}$
$\angle \mathrm{OCA}+\angle \mathrm{OAC}+\angle \mathrm{AOC}=180^{\circ}$
$2 \angle \mathrm{OAC}=180-170=10^{\circ}$
$\angle \mathrm{OAC}=5^{\circ}$

46. In the given figure, $\triangle\mathrm{ABC}$ and $\triangle\mathrm{DBC}$ are inscribed in a circle such that $\angle\mathrm{BAC}=60^{\circ}$ and $\angle\mathrm{DBC}=50^{\circ}$. Then $\angle\mathrm{BCD}=?$

(A) 50° (B) 60° (C) 70° (D) 80°



Ans. :

c. 70°

Solution:

Since angles in the same segment of a circle are equal.

$\angle\mathrm{BAC}=\angle\mathrm{BDC}=60^{\circ}$.

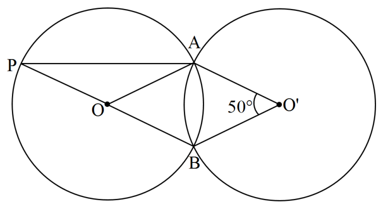
In $\triangle\mathrm{BDC}$,

$\angle\mathrm{BDC}+\angle\mathrm{DBC}+\angle\mathrm{BCD}=180^{\circ}$.

$\Rightarrow\ 60^{\circ}+50^{\circ}+\angle\mathrm{BCD}=180^{\circ}$

$\Rightarrow\ \angle\mathrm{BCD}=70^{\circ}$

47. The given figure shows two congruent circles with centre O and O’ intersecting at A and B. If $\angle\mathrm{AO'B}=50^{\circ}$, then the measure of $\angle\mathrm{APB}$ is:

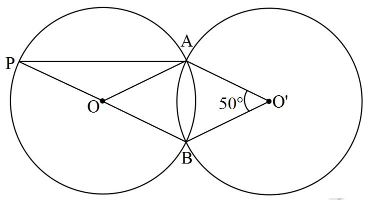


(A) 25º (B) 50º (C) 45º (D) 40º

Ans. :

a. 25º

Solution:



Since both the triangles are congruent,

So, OA = O'A,

OB = O'B

AB = AB (Common)

Hence, $\triangle\mathrm{AOB}\cong\triangle\mathrm{AO'B}$

Thus, $\angle\mathrm{AOB}=\angle\mathrm{AO'B}=50^{\circ}$

Since, PB is a straight line, therefore:-

$\angle\mathrm{AOP}+\angle\mathrm{AOB}=180^{\circ}$

$\Rightarrow\angle\mathrm{AOP}=180^{\circ}-50^{\circ}=130^{\circ}$

Again, In triangle OPA,

$\Rightarrow\angle\mathrm{P}=\angle\mathrm{A}$

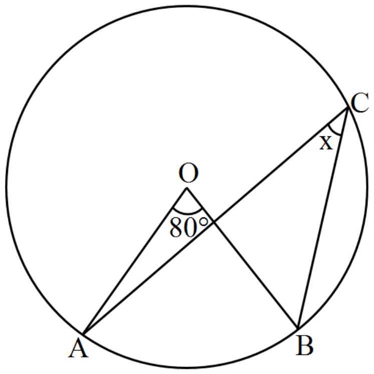
$\Rightarrow\angle\mathrm{A}+\angle\mathrm{P}+\angle\mathrm{O}=180^{\circ}$

$\Rightarrow 2\angle P + 130^\circ = 180^\circ$

$\Rightarrow \angle P = \frac{50^\circ}{2} = 25^\circ$

Thus, $\angle OPA = 25^\circ$

48. In the figure, O is the centre of eh circle and $\angle AOB = 80^\circ$. The value of x is:



(A) 40º (B) 30º (C) 160º (D) 60º

Ans. :

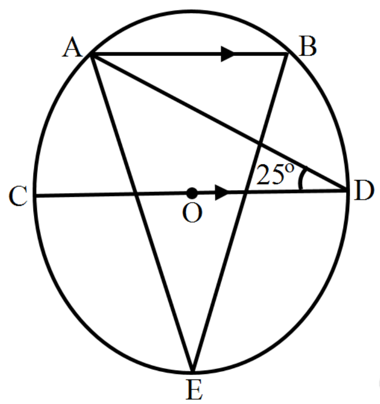
a. 40º

Solution:

Angle made by a chord at the centre is twice the angle made by it on any point on the circumference.

$x = \frac{\angle AOB}{2} = \frac{80^\circ}{2} = 40^\circ$

49. In the given figure, AB || CD and O is the centre of the circle. If $\angle ADC = 25^\circ$, then the measure of $\angle AEB$ is:

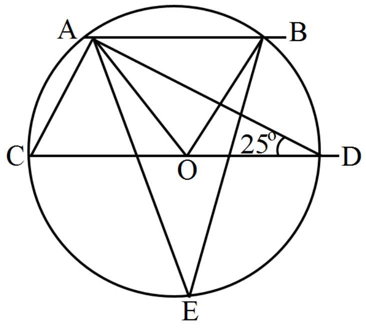


(A) 40º (B) 80º (C) 25º (D) 80º

Ans. :

a. 40º

Solution:



Here, AB || CD and $\angle ADC = 25^\circ$,

So, $\angle DAB = 25^\circ$, (opposite interior angles are equal)

Now, $\angle ADC = 25^\circ$, so, $\angle AOC = 50^\circ$ (Angle subtended by arc AC at centre is twice the angle subtended at circumference)

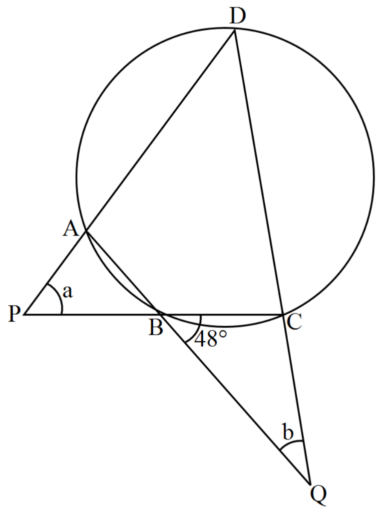
Similarly, $\angle DAB = 25^\circ$, so, $\angle DOB = 25^\circ$ (Angle subtended by arc BD at centre is twice the angle subtended at circumference)

$\angle AOB + \angle DOB + \angle AOC = 180^\circ$ (All lie in straight line)

$\angle AOB = 180 - 50 - 50 = 80^\circ$

Now, $\angle AEB = 40^\circ$ (Angle subtended by arc AB at centre is twice the angle subtended at circwnference)

50. In the given figure, ABCD is a cyclic quadrilateral, $\angle CBQ = 48^\circ$ and a = 2b. Then, b is equal to:

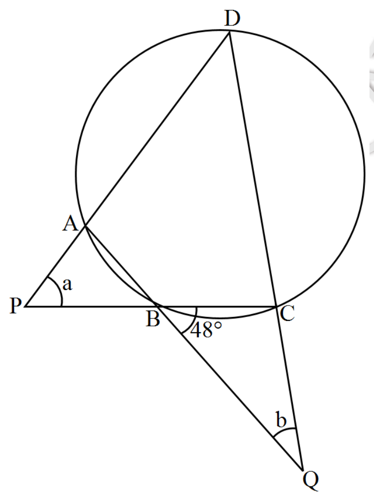


(A) 48º (B) 28º (C) 38º (D) 18º

Ans. :

b. 28º

Solution:



Here, $\angle ABC$ is supplementary to $\angle CBQ$

So, $\angle ABC = 180 - 48 = 132^\circ$.

Since, ABCD is cyclic quadrilateral, $\angle B + \angle D = 180^\circ$ {opposite angles are supplementery}

So, $\angle \mathrm{D} = 180 - 132 = 48^\circ$

Now, in triangle PDC, $\angle \mathrm{P} + \angle \mathrm{D} + \angle \mathrm{C} = 180^\circ$

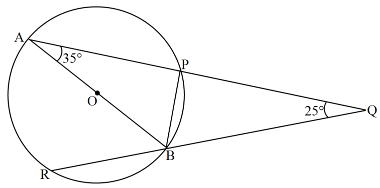
= a + 48º + (48º + b) = 180º {since, $\angle \mathrm{C}$ is external angle to B and b, and sum of two opposite interior angles is equal to external angle}

= 3b + 96 = 180º

3b = 180 - 96 = 84

b = 28º

51. In the given figure, AB is a diameter of the circle APBR. APQ and RBQ are straight lines. If $\angle \mathrm{A} = 35^\circ$ and then the measure of $\angle \mathrm{PBR}$ is:

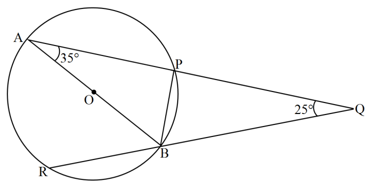


(A) 135º (B) 155º (C) 165º (D) 115º

Ans. :

d. 115º

Solution:



$\angle \mathrm{APB} = \angle \mathrm{BPQ} = 90^\circ$

Now,

In $\triangle \mathrm{APB}$,

$\angle \mathrm{BAP} + \angle \mathrm{APB} + \angle \mathrm{ABP} = 180^\circ$

$35^\circ + 90^\circ + \angle \mathrm{ABP} = 180^\circ$

$\angle \mathrm{ABP} = 55^\circ$

Again,

In $\triangle \mathrm{BPQ}$

$\Rightarrow \angle \mathrm{BPQ} + \angle \mathrm{PQB} + \angle \mathrm{PBQ} = 180^\circ$

$\Rightarrow 90^\circ + 25^\circ + \angle \mathrm{PBQ} = 180^\circ$

$\Rightarrow \angle \mathrm{PBQ} = 65^\circ$

Since, RBQ is a straight line,

$\angle \mathrm{RBA} + \angle \mathrm{ABP} + \angle \mathrm{PBQ} = 180^\circ$

$\angle \mathrm{RBA} + 55^\circ + 65^\circ = 180^\circ$

$\angle \mathrm{RBA} = 60^\circ$

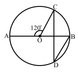
Finally,

$\angle \mathrm{PBR} = \angle \mathrm{ABP} + \angle \mathrm{RBA}$

$= 55^\circ + 60^\circ = 115^\circ$

52. In the given figure, O is the centre of a circle and $\angle \mathrm{AOC} = 120^\circ$. Then, $\angle \mathrm{BDC} = ?$

(A) 60° (B) 45° (C) 30° (D) 15°



Ans. :

c. 30°

Solution:

Since BOA is a diameter.

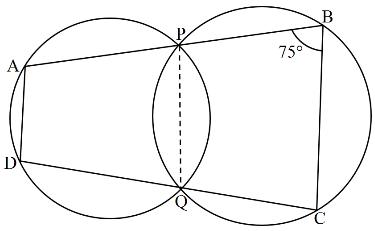
$\angle \mathrm{AOC} + \angle \mathrm{BOC} = 180^\circ$

$\Rightarrow\ 120^\circ + \angle \mathrm{BOC} = 180^\circ$

$\Rightarrow\ \angle \mathrm{BOC} = 60^\circ$

So, $\angle \mathrm{BDC} = \frac{1}{2}\angle \mathrm{BOC} = \frac{1}{2}(60^\circ) = 30^\circ$

53. The given figure shows two intersecting circles. If $\angle \mathrm{ABC} = 75^\circ$ then the measure of $\angle \mathrm{PAD}$ is:

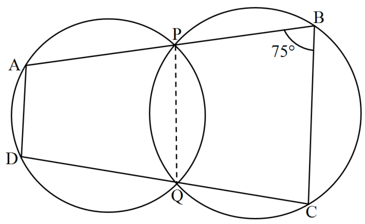


(A) 105º (B) 150º (C) 125º (D) 75º

Ans. :

a. 105º

Solution:



Since $\angle \mathrm{PQC} + \angle \mathrm{PBC} = 180^\circ$ (Opposite angles of a cyclic quadrilateral)

$\angle \mathrm{PQC} = 180^\circ - 75^\circ = 105^\circ$

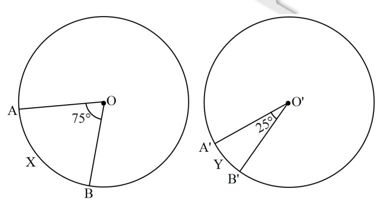
Again, $\angle \mathrm{DQP} + \angle \mathrm{PQC} = 180^\circ$ (Linear Pair)

So, $\angle \mathrm{DQP} = 75^\circ$

Also, $\angle \mathrm{PAD} + \angle \mathrm{DQP} = 180^\circ$ (Opposite angles of a cyclic quadrilateral)

$\angle \mathrm{PAD} = 105^\circ$

54. The given figures show two congruent circles with centre O and O’. Arc AXB subtends an angle of 75º at the centre and arc A’YB’ subtends an angle of 25º at the centre O’. Then, the ratio of arcs AXB to A ‘YB’ is:



(A) It is 2 : 1 (B) It is 3 : 1 (C) It is 1 : 3 (D) It is 1 : 2

Ans. :

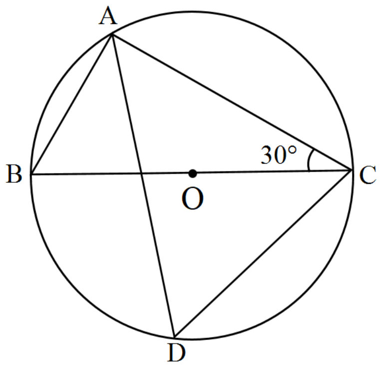
b. It is 3 : 1

Solution:

Since, circles are congruent thus we can consider the two arcs as in the same circle. Now the length of the arcs is directly proportional the angle subtended by the arcs. Therefore the lengths of the given arcs would be same in ratio as the ratio of the given angles.

Hence the required ratio is $\frac{75}{25} = \frac{3}{1}$.

55. In the given figure, BOC is a diameter of a circle with centre O. If $\angle \mathrm{BCA} = 30^\circ$, then $\angle \mathrm{CDA} = ?$



(A) 50º (B) 45º (C) 30º (D) 60º

Ans. :

d. 60º

Solution:

Angles in a semi-circle measure 90º.

$\therefore \angle \mathrm{BAC} = 90^\circ$

In $\triangle \mathrm{ABC}$, we have:

$\angle \mathrm{BAC} + \angle \mathrm{ABC} + \angle \mathrm{BCA} = 180^\circ$ (Angle sum property of a triangle)

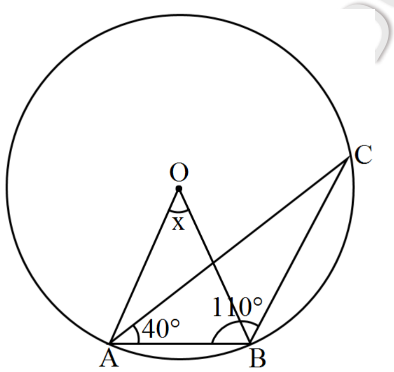
$\therefore 90^\circ + \angle \mathrm{ABC} + 30^\circ = 180^\circ$

$\Rightarrow \angle \mathrm{ABC} = (180^\circ - 120^\circ) = 60^\circ$

$\therefore \angle \mathrm{CDA} = \angle \mathrm{ABC} = 60^\circ$ (Angles in the same segment of a circle)

$\Rightarrow \angle \mathrm{CDA} = 60^\circ$

56. In the given figure, O is the centre of the circle. If $\angle \mathrm{CAB} = 40^\circ$ and $\angle \mathrm{CBA} = 110^\circ$, the value of x is:

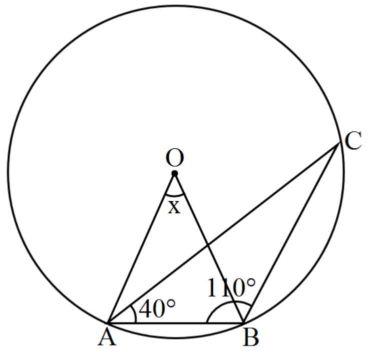


(A) 55º (B) 80º (C) 60º (D) 50º

Ans. :

c. 60º

Solution:



In $\triangle ABC$

$\angle A + \angle B + \angle C = 180^\circ$

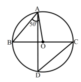
$\Rightarrow \angle C = 180^\circ - 40^\circ - 110 = 30^\circ$

Since AB is a chord and angle made by a chord at the centre is twice the angle made by it on any point on the circumference, therefore:-

x = 2 × 30º = 60º

57. In the given figure, O is the centre of a circle and $\angle OAB = 50^\circ$. Then, $\angle CDA = ?$

(A) 40° (B) 50° (C) 75° (D) 25°



Ans. :

b. 50°

Solution:

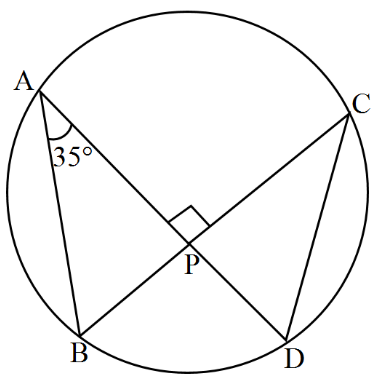
OA = OB [Radii of the same circle]

$\Rightarrow \ \angle OBA = \angle OAB = 50^\circ$

Since angles in the same segment are equal, $\angle ABC = \angle CDA$.

That is, $\angle ABO = \angle CDA = 50^\circ$

58. In a given figure, chords AD and BC intersect each other at right angles at point P. If $\angle DAB = 35^\circ$, then $\angle ADC =$



(A) 65º (B) 35º (C) 55º (D) 45º

Ans. :

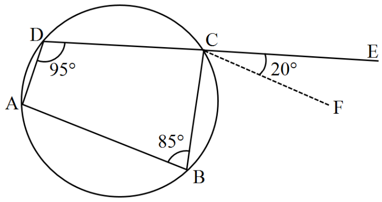
c. 55º

Solution:

From triangle APB, $\angle ABP = 180^\circ - 90^\circ - 35^\circ = 55^\circ$

Thus, $\angle ADC = 55^\circ (\angle ABC = \angle ADC)$

59. In the given figure, ABCD is a cyclic quadrilateral in which DC is produced to E and CF is drawn parallel to AB such that $\angle ADC = 95^\circ$ and $\angle ECF = 20^\circ$. Then, $\angle BAD = ?$



(A) 95º (B) 85º (C) 75º (D) 105º

Ans. :

d. 105º

Solution:

We have:

$\angle ABC + \angle ADC = 180^\circ$

$\Rightarrow \angle ABC + 95^\circ = 180^\circ$

$\Rightarrow \angle ABC = (180^\circ - 95^\circ) = 85^\circ$

Now, CF || AB and CB is the transversal.

$\therefore \angle BCF = \angle ABC = 85^\circ$ (Alternate interior angles)

$\Rightarrow \angle BCE = (85^\circ + 20^\circ) = 105^\circ$

$\Rightarrow \angle DCB = (180^\circ - 105^\circ) = 75^\circ$

$\Rightarrow \angle DCB = 75^\circ$

Now, $\angle BAD + \angle BCD = 180^\circ$

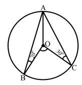
$\Rightarrow \angle BAD + 75^\circ = 180^\circ$

$\Rightarrow \angle BAD = (180^\circ - 75^\circ)$

$\Rightarrow \angle BAD = 105^\circ$

60. In the given figure, O is the centre of a circle in which $\angle OBA = 20^\circ$ and $\angle OCA = 30^\circ$. Then, $\angle BOC = ?$

(A) 50° (B) 90° (C) 100° (D) 130°



Ans. :

c. 100°

Solution:

In $\triangle OAB$,

OA = OB [Radii of the same circle]

$\Rightarrow \angle OBA = \angle OAB = 20^\circ$ [Angle opposite equal sides are equal]

In $\triangle OAC$,

OA = OC [Radii of the same circle]

$\Rightarrow \angle OCA = \angle OAC = 30^\circ$ [Angle opposite equal sides are equal]

Now, $\angle BAC = \angle BAO + \angle CAO$

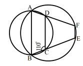
$= 20^\circ + 30^\circ$

$= 50^\circ$

$\angle BOC = 2\angle BAC = 2(50^\circ) = 100^\circ.$

61. In the given figure, ABCD and ABEF are two cyclic quadrilaterals. If $\angle BCD = 110^\circ$ then $\angle BEF = ?$

(A) 55° (B) 70° (C) 90° (D) 110°



Ans. :

d. 110°

Solution:

Since ABCD is a cyclic qyadrilateral, we have:

$\angle BAD + \angle BCD = 180^\circ$

$\Rightarrow \angle BAD + 110^\circ = 180^\circ$

$\Rightarrow \angle BAD = 70^\circ$

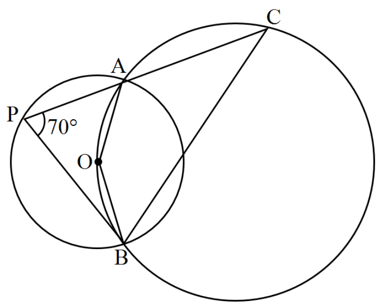
Since ABEF is a cyclic qyadrilateral, we have:

$\angle BAD + \angle BEF = 180^\circ$

$\Rightarrow 70^\circ + \angle BEF = 180^\circ$

$\Rightarrow \angle BEF = 110^\circ$

62. The figure shows two circles which intersect at A and B. The centre of the smaller circle is O and it lies on the circumference of the larger circle. If $\angle APB = 70^\circ$, then the measure of $\angle ACB$ is:

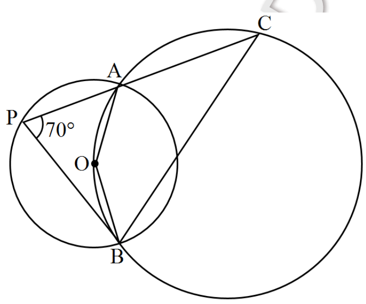


(A) 40º (B) 50º (C) 60º (D) 70º

Ans. :

a. 40º

Solution:



Since, AB is a chord and makes $\angle APB = 70^\circ$ at the circumference, so $\angle AOB = 140^\circ$

Now, as AOBC is a cyclic quadrilateral then, sum of opposite angles must be 180º

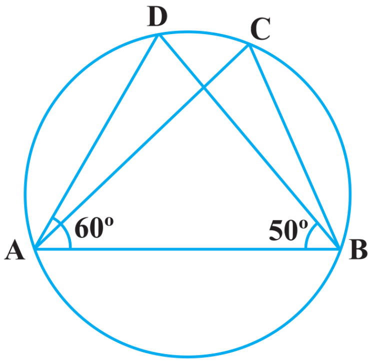
$\angle AOB + \angle ACB = 180^\circ$

$\Rightarrow 140^\circ + \angle ACB = 180^\circ$

$\Rightarrow \angle ACB = 40^\circ$

63. Write the correct answer in the following:

In Fig. if $\angle DAB = 60^\circ, \angle ABD = 50^\circ$, then $\angle ACB$ is equal to:



(A) 60°. (B) 50°. (C) 70°. (D) 80°.

Ans. :

c. 70°.

Solution:

In $\triangle ADB$, we have

$\angle A + \angle B + \angle D = 180^\circ$ [Angle sum property of a triangle]

$\Rightarrow 60^\circ + 50^\circ + \angle D = 180^\circ$

$\Rightarrow \angle D = 180^\circ - 110^\circ = 70^\circ$

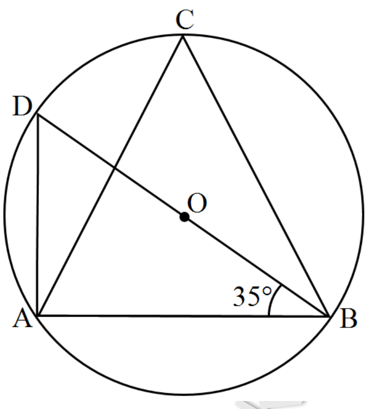
i.e., $\angle ABD = 70^\circ$

Now, $\angle ACB = \angle ADB = 70^\circ$

[$\because$ Angles in the same segment of a circle are equal]

Hence, (c) is the correct answer.

64. In the given figure, O is the centre of the circle. If $\angle DBA = 35^\circ$, then the measure of $\angle ACB$ is equal to:

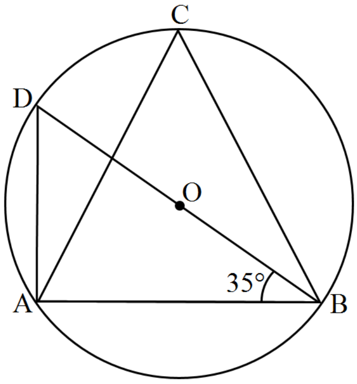


(A) 60º (B) 55º (C) 65º (D) 45º

Ans. :

b. 55º

Solution:

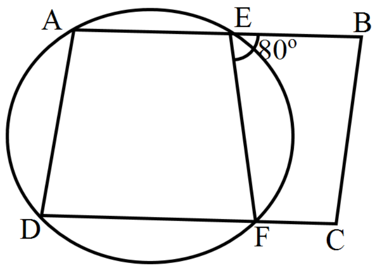


Join OA.

Now, in triangle AOB, from angle sum property we can find that $\angle \mathrm{AOB} = 110^{\circ}$

Now, $2\angle \mathrm{ACB} = \angle \mathrm{AOB} = \frac{110^{\circ}}{2} = 55^{\circ}$

65. ABCD is a parallelogram. A circle passes through A and D and cuts AB at E and DC at F. If $\angle \mathrm{BEF} = 80^{\circ}$, then $\angle \mathrm{ABC}$ is equal to:

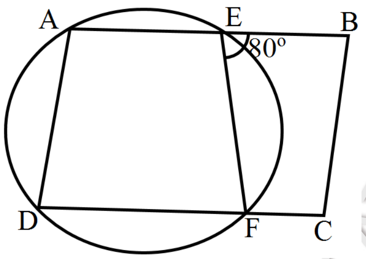


(A) 80º (B) 75º (C) 120º (D) 100º

Ans. :

a. 80º

Solution:



$\angle \mathrm{AEF} + 80^{\circ} = 180^{\circ}$

$\angle \mathrm{AEF} = 100^{\circ}$

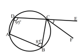
$\angle \mathrm{ADF} + \angle \mathrm{AEF} = 180^{\circ}$ (Opposite angles of a cyclic quadrilateral)

$\angle \mathrm{ADF} = 180^{\circ} - 100^{\circ} = 80^{\circ}$

$\angle \mathrm{ADF} = \angle \mathrm{ABC} = 80^{\circ}$ (opposite angles of a parallelogram)

66. In the given figure, ABCD is a cyclic quadrilateral in which DC is produced to E and CF is drawn parallel to AB such that $\angle \mathrm{ADC} = 95^{\circ}$ and $\angle \mathrm{ECF} = 20^{\circ}$. Then, $\angle \mathrm{EAD} = ?$

(A) 95° (B) 85° (C) 105° (D) 75°



Ans. :

c. 105°

Solution:

Since CF || AB, $\angle \mathrm{ABC} = \angle \mathrm{BCF} = 85^{\circ}$

$\angle \mathrm{BAD} = \angle \mathrm{BCE}$

$\Rightarrow \angle \mathrm{BAD} = \angle \mathrm{BCF} + \angle \mathrm{ECF}$

$\Rightarrow \angle \mathrm{BAD} = 105^{\circ}$

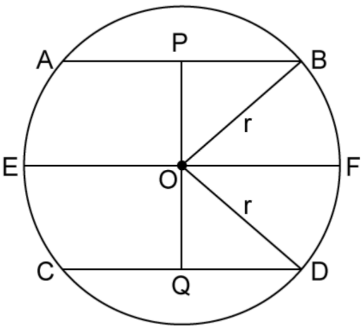
67. In a circle of radius 17cm, two parallel chords are drawn on opposite side of a diameter. The distance between the chords is 23cm. If the length of one chord is 16cm, then the length of the other is:

(A) 34cm. (B) 15cm. (C) 23cm. (D) 30cm.

Ans. :

d. 30cm.

Solution:



PQ = 23cm

AB = 16cm

⇒ BP = AP = 8cm

r = 17cm

⇒ EF = diameter = 2r = 34cm

Consider $\triangle \mathrm{OPB}$,

$r^2 = OP^2 + BP^2$

$\Rightarrow OP^2 = (17)^2 - (8)^2 = 289 - 64 = 225$

⇒ OP = 15cm

⇒ OQ = 23 - 15 = 8cm

Consider $\triangle \mathrm{OQD}$,

$r^2 = OQ^2 + QD^2$

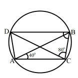
$\Rightarrow QD^2 = r^2 - OQ^2 = (17)^2 - (8)^2 = 225$

⇒ OD = 15cm

⇒ CD = 2 × QD = 30cm

68. In the give figure, AB and CD are two intersecting chords of a circle. If $\angle \mathrm{CAB} = 40^{\circ}$ and $\angle \mathrm{BCD} = 80^{\circ}$ then $\angle \mathrm{CBD} = ?$

(A) 80° (B) 60° (C) 50° (D) 70°



Ans. :

b. 60°

Solution:

Since angles in the same segment are equal,

$\angle BDC = \angle BAC = 40^\circ$

In $\triangle BDC$,

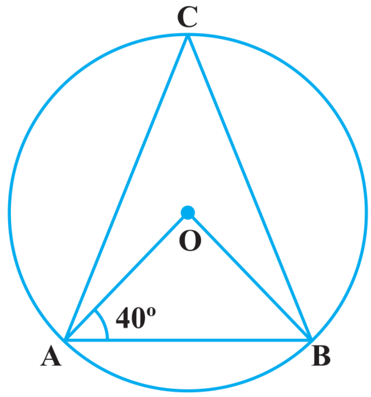
$\angle BDC + \angle BCD + \angle CBD = 180^\circ$ [Angle sum property]

$\Rightarrow\ 40^\circ + 80^\circ + \angle CBD = 180^\circ$

$\Rightarrow\ \angle CBD = 60^\circ$

69. Write the correct answer in the following:

In Fig. if $\angle OAB = 40^\circ$, then $\angle ACB$ is equal to:



(A) 50°. (B) 40°. (C) 60°. (D) 70°.

Ans. :

a. 50°.

Solution:

In $\triangle QAB$, $OA = OB$ [both are the radius of a circle]

$\angle OAB = \angle OBA \Rightarrow \angle OBA = 40^\circ$

[angles opposite to equal sides are equal]

Also, $\angle AOB = \angle OBA \Rightarrow \angle BAO = 180^\circ$

[by angle sum property of a triangle]

$\angle AOB + 40^\circ + 40^\circ = 180^\circ$

$\Rightarrow\ \angle AOB = 180^\circ - 80^\circ = 100^\circ$

We know that, in a circle, the angle subtended by an arc at the centre is twice the angle subtended by it at the remaining part of the circle.

$\angle AOB = 2\angle ACB \Rightarrow 100^\circ = 2\angle ACB$

$\angle ACB = \frac{100}{2} = 50^\circ$

70. Greatest chord of a circle is called its:

(A) Chord (B) Diameter (C) Secant (D) Radius

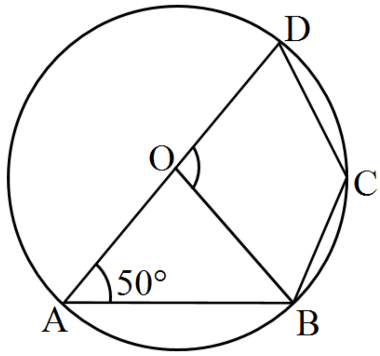
Ans. :

b. Diameter

Solution:

Since diameter is the longest segment that can be drawn in a circle(touching the circle at both ends), therefore it is the longest possible chord also.

71. In the given figure, O is the centre of a circle and $\angle OAB = 50^\circ$. Then, $\angle BOD = ?$



(A) 80º (B) 100º (C) 130º (D) 50º

Ans. :

b. 100º

Solution:

OA = OB (Radii of a circle)

$\Rightarrow \angle OBA = \angle OAB = 50^\circ$

In $\triangle OAB$, we have:

$\angle OAB + \angle OBA + \angle AOB = 180^\circ$ (Angle sum property of a triangle)

$\Rightarrow 50^\circ + 50^\circ + \angle AOB = 180^\circ$

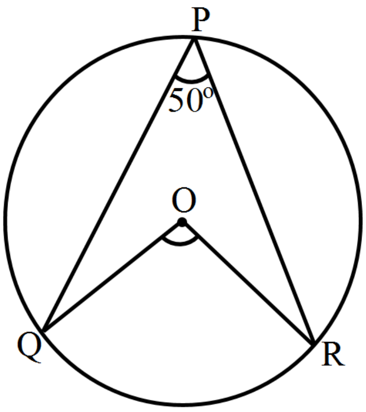
$\Rightarrow \angle AOB = (180^\circ - 100^\circ) = 80^\circ$

Since $\angle AOB + \angle BOD = 180^\circ$ (Linear pair)

$\therefore \angle BOD = (180^\circ - 80^\circ) = 100^\circ$

$\Rightarrow \angle BOD = 100^\circ$

72. In the given figure, O is the centre of the circle. If $\angle QPR$ is 50º, then $\angle QOR$ is:



(A) 100º (B) 130º (C) 40º (D) 50º

Ans. :

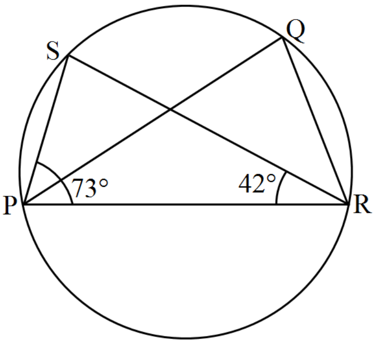
a. 100º

Solution:

Angle made by a chord at the centre is twice the angle made by it on any point on the circumference. Therefore,

$\angle QOR = 2\angle QPR = 50^\circ \times 2 = 100^\circ$

73. In the figure, if $\angle SPR = 73^\circ, \angle SRP = 42^\circ$ then $\angle PQR$ is equal to:



(A) 74º (B) 76º (C) 65º (D) 70º

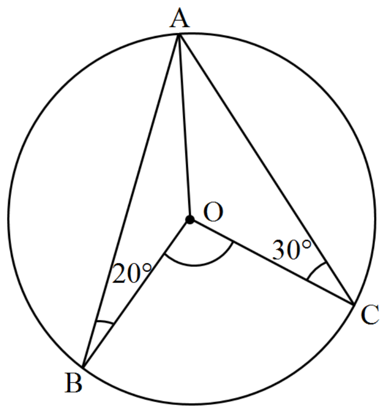
Ans. :

c. 65º

Solution:

$\angle PQR = \angle PSR = 180^\circ - 73^\circ - 42^\circ = 65^\circ$

74. In the given figure, O is the centre of a circle in which $\angle OBA = 20^\circ$ and $\angle OCA = 30^\circ$. Then, $\angle BOC = ?$



(A) 130º (B) 90º (C) 50º (D) 100º

Ans. :

d. 100º

Solution:

In $\triangle OAB$, we have:

OA = OB (Radii of a circle)

$\Rightarrow \angle OAB = \angle OBA = 20^\circ$

In $\triangle OAC$, we have:

OA = OC (Radii of a circle)

$\Rightarrow \angle OAC = \angle OCA = 30^\circ$

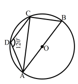
Now, $\angle BAC = (20^\circ + 30^\circ) = 50^\circ$

$\therefore \angle BOC = (2 \times \angle BAC) = (2 \times 50^\circ) = 100^\circ$

$\Rightarrow \angle BOC = 100^\circ$

75. In the given figure, AOB is a diameter and ABCD is a cyclic quadrilateral. If $\angle ADC = 120^\circ$ then $\angle BAC = ?$

(A) 60° (B) 30° (C) 20° (D) 45°



Ans. :

b. 30°

Solution:

We know that the opposite angles of a quadrilateral are supplementary.

$\angle \mathrm{ADC}+\angle \mathrm{ABC}=180^{\circ}$

$\Rightarrow \ 120^{\circ}+\angle \mathrm{ABC}=180^{\circ}$

$\Rightarrow \ \angle \mathrm{ABC}=60^{\circ}$

Since BOC is a diameter $\angle \mathrm{ACB}=90^{\circ}$.

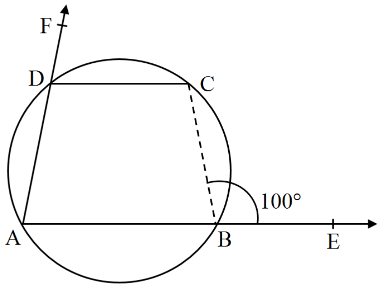
In $\triangle \mathrm{CAB}$,

$\angle \mathrm{ABC}+\angle \mathrm{BAC}+\angle \mathrm{ACB}=180^{\circ}$ [Angle sum property]

$\Rightarrow \ 60^{\circ}+\angle \mathrm{BAC}+90^{\circ}=180^{\circ}$

$\Rightarrow \ \angle \mathrm{BAC}=30^{\circ}$

76. In the given figure, sides AB and AD of quad. ABCD are produced to E and F respectively. If $\angle \mathrm{CBE}=100^{\circ}$, then $\angle \mathrm{CDE}=?$



(A) 130º (B) 100º (C) 80º (D) 90º

Ans. :

c. 80º

Solution:

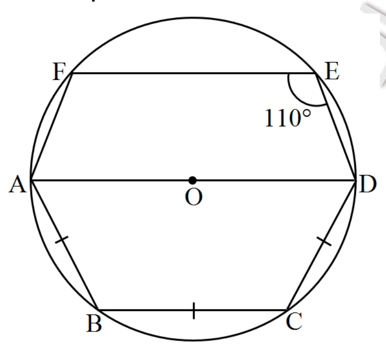
In a cyclic quadrilateral ABCD, we have:

Interior opposite angle, $\angle \mathrm{ADC}=\text{exterior } \angle \mathrm{CBE}=100^{\circ}$

$\therefore \angle \mathrm{CDF}=\left(180^{\circ}-\angle \mathrm{ADC}\right)=\left(180^{\circ}-100^{\circ}\right)=80^{\circ}$ (Linear pair)

$\Rightarrow \angle \mathrm{CDF}=80^{\circ}$

77. In the given figure, AD is a diameter of the circle with centre O. Chords AB, BC and CD are equal. If $\angle \mathrm{DEF}=110^{\circ}$, then $\angle \mathrm{FAB}$ is equal to:

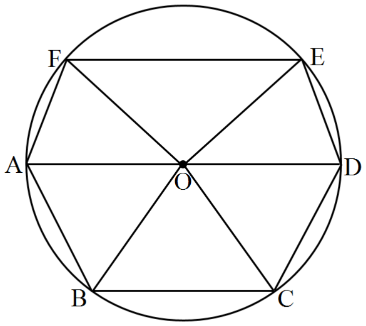


(A) 130º (B) 110º (C) 140º (D) 120º

Ans. :

a. 130º

Solution:



Here, given AB = BC = CD

Now, equal chords subtend equal angles at centre. So, $\angle AOC = \angle BOC = \angle COD$

Also, they lie in straight line so, $\angle AOC + \angle BOC + \angle COD = 180^\circ$

$\angle AOB = \angle BOC = \angle COD = 60^\circ$

In $\triangle AOB$

$AO = OB, \angle OAB = \angle OBA$

Since, $\angle AOB = 60^\circ, \angle OAB = \angle OBA = 60^\circ$

Now, $\angle DOE = \angle AOB = 60^\circ$ (vertically opposite angle)

In $\triangle DOE, OD = OE$ (radius)

so, $\angle ODE = \angle OED$

$\triangle DOE, \angle DOE + \angle ODE + \angle OED = 180^\circ$

$2\angle ODE = \angle OED = 180 - 60 = 120^\circ$

$\angle ODE = \angle OED = 60^\circ$

given was, $\angle DEF = 110^\circ$, so, $\angle OEF = 110 - 60 = 50^\circ$

Now, in $\triangle EOF$ $OE = OF$

so, $\angle OEF = \angle OFE = 50^\circ$

In, $\triangle EOF$ $\angle FOE = 180 - (50 + 50) = 80^\circ$

Now, $\angle DOE + \angle FOE + \angle AOF = 180^\circ$ (All lie on same straight line)

So, $\angle AOF = 180 - (80 + 60) = 40^\circ$

Now, in $\triangle AOF$ $AO = FO$

SO, $\angle OFA = \angle OAF$

In $\triangle AOF, 2\angle OAF + \angle FOA = 180^\circ$

$2\angle OAF + \angle FOA = 180^\circ$

$\angle OAF = 90 - 20 = 70^\circ$

So, $\angle FAB = \angle FAO + \angle OAB$

$= 70^\circ + 60^\circ = 130^\circ$

78. If ABC is an arc of a circle and $\angle ABC = 135^\circ$, then the ratio of arc ABC^ to the circumference, is:

(A) 1 : 4 (B) 1 : 2 (C) 3 : 8 (D) 3 : 4

Ans. :

c. 3 : 8

Solution:

The length of an arc subtending an angle θ in a circle of radius r is given by the formula, Length of the arc $= \frac{\theta}{360^\circ} 2\pi r$

Here, it is given that the are subtends an angle of 135º with its centre. So the length of the given arc in a circle with radius r is given as

Length of the arc $= \frac{135^\circ}{360^\circ} 2\pi r \ldots . (i)$

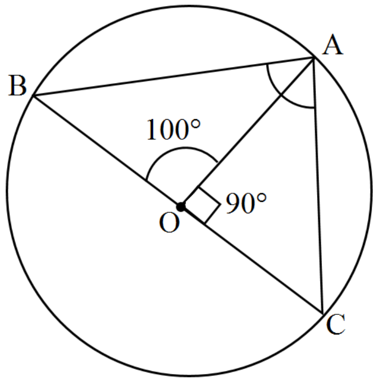
The circumference of the same circle with radius $r = 2\pi r. \ \ldots . (ii)$

The ratio between the lengths of the arc and the circumference of the circle will be

$\frac{\text{Length of the arc}}{\text{Cirrumference of the circle}} = \frac{135^\circ(2\pi r)}{360^\circ(2\pi r)} = \frac{135^\circ}{360^\circ} = \frac{3}{8}$ [FROM (i) and (ii)]

RATIO = 3 : 8

79. In the given figure, O is the centre of a circle. If $\angle AOB = 100^\circ$ and $\angle AOC = 90^\circ$, then $\angle BAC = ?$



(A) 95º (B) 85º (C) 75º (D) 80º

Ans. :

b. 85º

Solution:

We have:

$\angle BOC + \angle BOA + \angle AOC = 360^\circ$

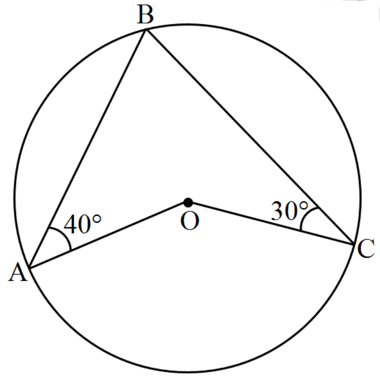
$\Rightarrow \angle BOC + 100^\circ + 90^\circ = 360^\circ$

$\Rightarrow \angle BOC = (360^\circ - 190^\circ) = 170^\circ$

$\therefore \angle BAC = (\frac{1}{2} \times \angle BOC) = (\frac{1}{2} \times 170^\circ) = 85^\circ$

$\Rightarrow \angle BAC = 85^\circ$

80. In the given figure, O is the centre of the circle. $\angle OAB$ and $\angle OCB$ are 40º and 30º respectively. Then, the measure of $\angle AOC$ is:

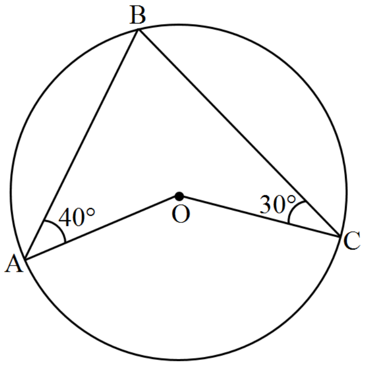


(A) 120º (B) 170º (C) 110º (D) 140º

Ans. :

d. 140º

Solution:



OA = OB = OC = Radius

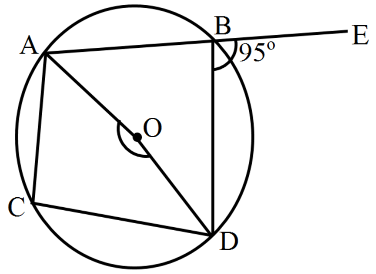
So, $\angle OAB \angle ABO = 40^\circ$

and, $\angle OCB = \angle ABO = 30^\circ$

Thus, $\angle ABC = 30 + 40 = 70^\circ$

So, $\angle AOC = 70 \times 2 = 140^\circ$ {Angle subtended by arc at centre is twice of angle subtended at circumference}

81. In the given figure, O is the centre of the circle ABE is a straight line. If $\angle DBE = 95^\circ$, then $\angle AOD$ is equal to:

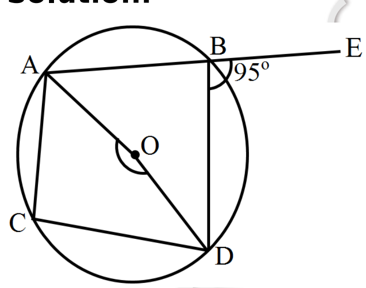


(A) 170º (B) 180º (C) 190º (D) 175º

Ans. :

a. 170º

Solution:

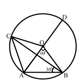


$\angle ABD = 180^\circ - 95 = 85^\circ$ (Linear Pair)

Since, $\angle AOD = 2\angle ABD = 2 \times 85^\circ = 170^\circ$

82. In the given figure, $\angle AOB = 90^\circ$ and $\angle ABC = 30^\circ$. Then, $\angle CAO = ?$

(A) 30° (B) 45° (C) 60° (D) 90°



Ans. :

c. 60°

Solution:

$\angle AOB = 2\angle ACB$

$\Rightarrow\ \angle ACB = \frac{1}{2}\angle AOB$

$\Rightarrow\ \angle ACB = \frac{1}{2}(90^\circ)$

$\Rightarrow\ \angle ACB = 45^\circ$

$\angle COA = 2\angle CBA = 2(30^\circ) = 60^\circ$

Since AOD is a straight line,

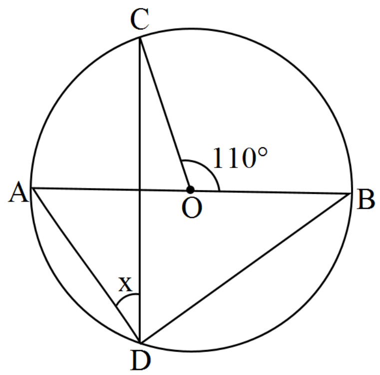
$\therefore\ \angle COD + \angle AOC = 180^\circ$

$\therefore\ \angle COD + 60^\circ = 180^\circ$

$\therefore\ \angle COD = 120^\circ$

$\Rightarrow\ \angle CAO = \frac{1}{2}\angle COD = \frac{1}{2} \times 120^\circ = 60^\circ$

83. The value of x in the given figure is:

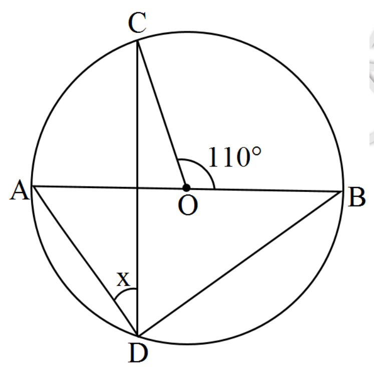


(A) 30º (B) 45º (C) 35º (D) 25º

Ans. :

c. 35º

Solution:

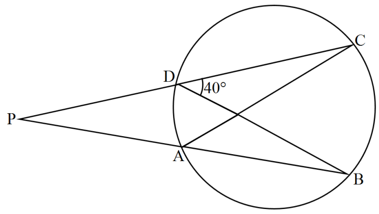


$\angle CDB = \frac{110^\circ}{2} = 55^\circ$

Now, $\angle ADB = 90^\circ$ (Angle in a semicircle)

So, $\angle ADC = x = 90^\circ - 55^\circ = 35^\circ$

84. In the given figure, if $\angle CDB = 40^\circ$, then the measure of $\angle PAC$ is:

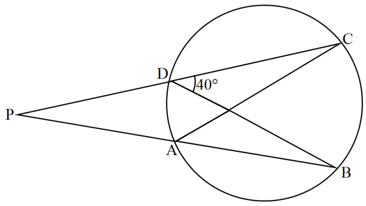


(A) 120º (B) 140º (C) 160º (D) 100º

Ans. :

b. 140º

Solution:



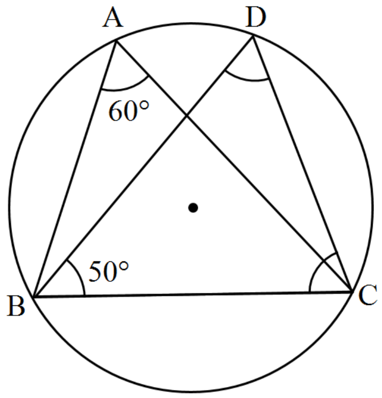
Since $\angle \mathrm{CDB} = \angle \mathrm{CAB}$

So, $\angle \mathrm{CAB} = 40^\circ$

Now $\angle \mathrm{PAC} + \angle \mathrm{CAB} = 180^\circ$ [Linear Pair]

Hence, $\angle \mathrm{PAC} = 14^\circ$

85. In the given figure, $\triangle \mathrm{ABC}$ and $\triangle \mathrm{DBC}$ are inscribed in a circle such that $\angle \mathrm{BAC} = 60^\circ$ and $\angle \mathrm{DBC} = 50^\circ$. Then, $\angle \mathrm{BCD} = ?$



(A) 60º (B) 50º (C) 80º (D) 70º

Ans. :

d. 70º

Solution:

$\angle \mathrm{BDC} = \angle \mathrm{BAC} = 60^\circ$ (Angles in the same segment of a circle)

In $\triangle \mathrm{BDC}$, we have

$\angle \mathrm{DBC} + \angle \mathrm{BDC} + \angle \mathrm{BCD} = 180^\circ$ (Angle sum property of a triangle)

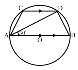
$\therefore 50^\circ + 60^\circ + \angle \mathrm{BCD} = 180^\circ$

$\Rightarrow \angle \mathrm{BCD} = 180^\circ - (50^\circ + 60^\circ) = (180^\circ - 110^\circ) = 70^\circ$

$\Rightarrow \angle \mathrm{BCD} = 70^\circ$

86. In the given figure, AOB is a diameter of a circle and CD || AB. If $\angle \mathrm{BAD} = 30^\circ$ then $\angle \mathrm{CAD} = ?$

(A) 30° (B) 60° (C) 45° (D) 50°



Ans. :

a. 30°

Solution:

Since AB || CD, $\angle\mathrm{BAD} = \angle\mathrm{CDA} = 30^\circ$ [Alternate angles]

Since AOB is a diameter, $\angle\mathrm{ADB} = 90^\circ$

$\angle\mathrm{CDB} = \angle\mathrm{CDA} + \angle\mathrm{ADB} = 30^\circ$

$\Rightarrow\ \angle\mathrm{CDB} = 30^\circ + 90^\circ$

$\Rightarrow\ \angle\mathrm{CDB} = 120^\circ$

We know that the opposite angles of a quadrilateral are supplementary.

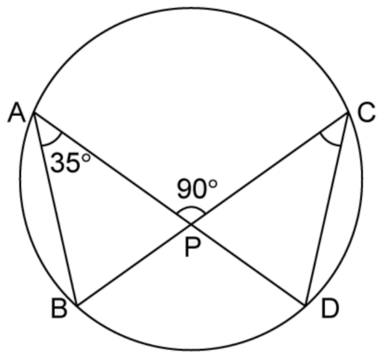
$\angle\mathrm{CAB} + \angle\mathrm{CDB} = 180^\circ$

$\Rightarrow\ \angle\mathrm{CAD} + \angle\mathrm{DAB} + \angle\mathrm{CDB} = 180^\circ$

$\Rightarrow\ \angle\mathrm{CAD} + 30^\circ + 120^\circ = 180^\circ$

$\Rightarrow\ \angle\mathrm{CAD} = 30^\circ$

87. In the given figure, chords AD and BC intersect each other at right angles at a point P. If $\angle\mathrm{DAB} = 35^\circ$, then $\angle\mathrm{ADC} =$

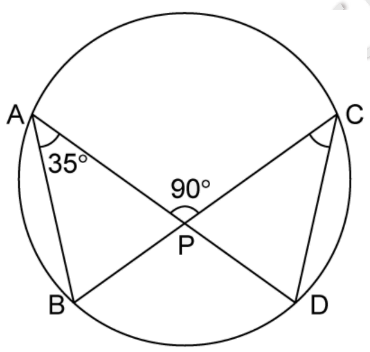


(A) 35° (B) 45° (C) 55° (D) 65°

Ans. :

c. 55°

Solution:



$\angle\mathrm{APC} + \angle\mathrm{APB} = 180^\circ$

$\Rightarrow \angle\mathrm{APB} = 180^\circ - 90^\circ = 90^\circ$

In $\triangle\mathrm{APB}$,

$\angle\mathrm{ABP} = 180^\circ - \angle\mathrm{APB} - \angle\mathrm{BAP}$

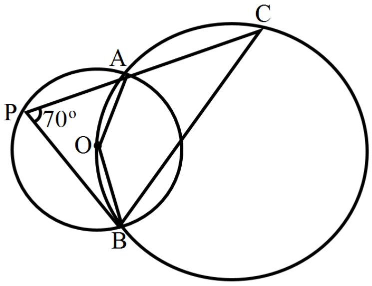
$180^\circ - 90^\circ - 35^\circ = 55^\circ$

Now Arc $\widehat{AC}$ makes $\angle ABC$ and $\angle ADC$ on circle.

$\Rightarrow ABC = \angle ADC$

$\Rightarrow \angle ADC = 55^\circ$

88. The figure shows two circles which intersect at A and B. The centre of the smaller circle is O and it lies on the circumference of the larger circle. If $\angle APB = 70^\circ$, then the measure of $\angle AGB$ is:

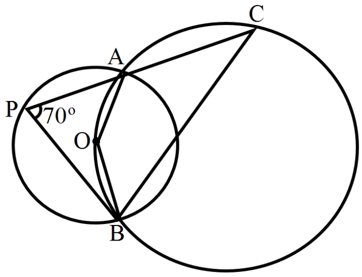


(A) 40º (B) 50º (C) 70º (D) 50º

Ans. :

a. 40º

Solution:



Since, AB is a chord and makes $\angle APB = 70^\circ$ at the circumference, so $\angle AOB = 140^\circ$

Now, as AOBC is a cyclic quadrilateral then, sum of opposite angles must be 180º.

$\angle AOB + \angle ACB = 180^\circ$

$\Rightarrow 140^\circ + \angle ACB = 180^\circ$

$\Rightarrow \angle ACB = 40^\circ$

89. An equilateral triangle ABC is inscribed in a circle with centre O. The measures of $\angle BOC$ is:

(A) 60º (B) 90º (C) 30º (D) 120º

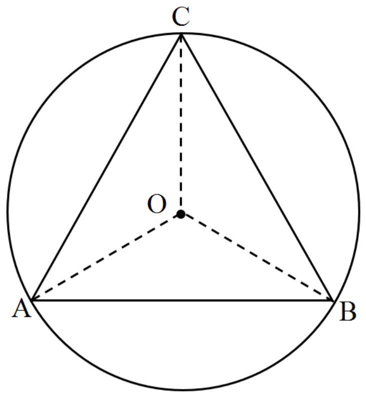
Ans. :

d. 120º

Solution:

We are given that an equilateral $\triangle ABC$ is inscribed in a circle with centre O. We need to find $\angle BOC$.

We have the following corresponding figure.



We are given AB = BC = AC

Since the sides, AB, BC, and AC are these equal chords of the circle.

Hence,

$\angle \mathrm{AOB} + \angle \mathrm{BOC} + \angle \mathrm{AOC} = 360$

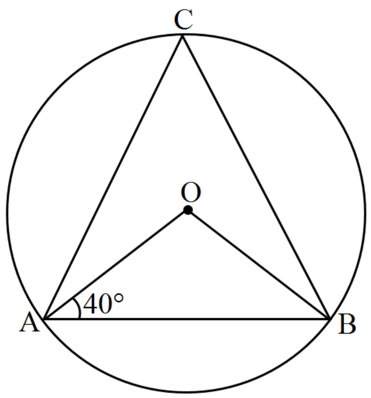
$\Rightarrow \angle \mathrm{BOC} + \angle \mathrm{BOC} + \angle \mathrm{BOC} = 360$

$\Rightarrow 3\angle \mathrm{BOC} = 360$

$\Rightarrow \angle \mathrm{BOC} = \frac{360}{3}$

$\Rightarrow \angle \mathrm{BOC} = 120^{\circ}$

90. In the figure, O is the center of the circle. If $\angle \mathrm{OAB} = 40^{\circ}$, then $\angle \mathrm{ACB}$ is equal to:

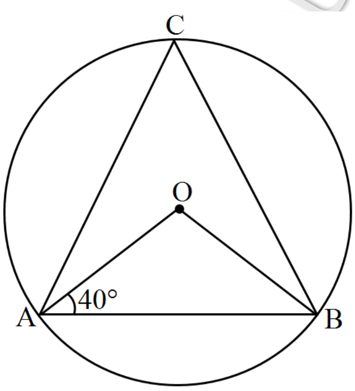


(A) 60º (B) 70º (C) 40º (D) 50º

Ans. :

d. 50º

Solution:



In $\triangle \mathrm{AOB}$

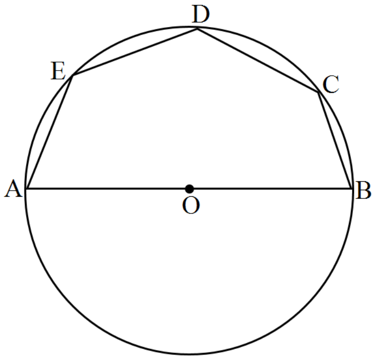
$\angle \mathrm{A} = \angle \mathrm{B} = 40^{\circ}$

And $\angle A + \angle B + \angle O = 180^\circ$

$\Rightarrow \angle O = 100^\circ$

So, $\angle ACB = \frac{100^\circ}{2} = 50^\circ$

91. AOB is a diameter of the circle and C, D, E are any three points on the semicircle. Then $\angle AED + \angle BCD$ is equal to:

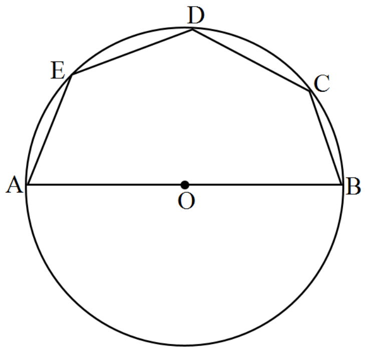


(A) 260º (B) 280º (C) 250º (D) 270º

Ans. :

d. 270º

Solution:



Join OE, OD, OC.

We get four triangles, namely, $\triangle AOE, \triangle EOD, \triangle DOC, \triangle COB$

Now, all these triangles are isosceles triangles.

So,

$\angle 1 = \angle 2$

$\angle 3 = \angle 4$

$\angle 5 = \angle 6$

$\angle 7 = \angle 8$

Now, required angle $\angle AED + \angle BCD = \angle 2 + \angle 3 + \angle 6 + \angle 7$

Now sum of all the angles must be 720º (as they are the angles of four triangles $\therefore \text{Sum} = 180^\circ \times 4 = 720^\circ$)

Therefore,

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 + \angle 9 + \angle 10 + \angle 11 + \angle 12$
$= 720^\circ$

$\Rightarrow 2\angle 2 + 2\angle 3 + 2\angle 6 + 2\angle 7 + \angle 9 + \angle 10 + \angle 11 + \angle 12 = 720^\circ$

$\Rightarrow 2(\angle 2 + \angle 3 + \angle 6 + \angle 7) + \angle 9 + \angle 10 + \angle 11 + \angle 12 = 720^\circ$

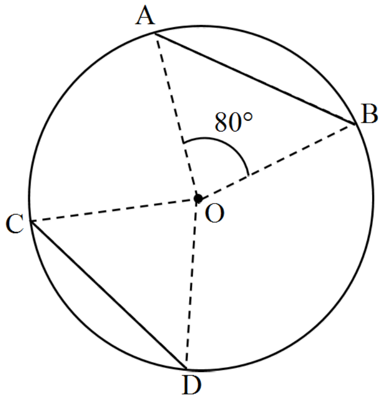
$\Rightarrow 2(\angle 2 + \angle 3 + \angle 6 + \angle 7) + 180^\circ = 720$

$\Rightarrow 2(\angle 2+\angle 3+\angle 6+\angle 7)=540^\circ$

$\Rightarrow (\angle 2+\angle 3+\angle 6+\angle 7)=\frac{540^\circ}{2}=270^\circ$

Thus, the required angle is equal to 270º

92. AB and CD are two equal chords of a circle with centre O such that $\angle \mathrm{AOB}=80^\circ$, then $\angle \mathrm{COD}=?$



(A) 100º (B) 80º (C) 120º (D) 40º

Ans. :

b. 80º

Solution:

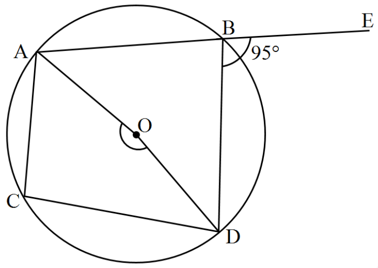
Given: AB = CD

We know that equal chords of a circle subtend equal angles at the centre.

$\therefore \angle \mathrm{COD}=\angle \mathrm{AOB}=80^\circ$

$\Rightarrow \angle \mathrm{COD}=80^\circ$

93. In the given figure, O is the centre of the circle ABE is a straight line. if $\angle \mathrm{DBE}=95^\circ$ then $\angle \mathrm{AOD}$ is equal to:

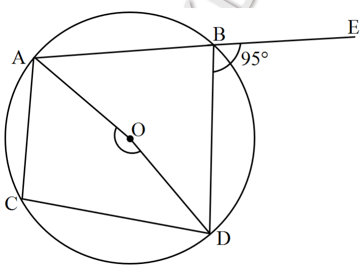


(A) 170º (B) 175º (C) 180º (D) 190º

Ans. :

a. 170º

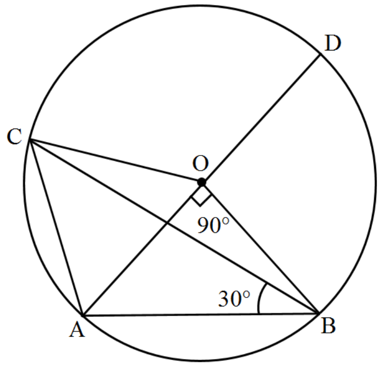
Solution:



$\angle \mathrm{ABD}=180^\circ-95^\circ=85^\circ$ (Linear Pair)

Since, $\angle \mathrm{AOD}=2\angle \mathrm{ABD}=2\times 85^\circ=170^\circ$

94. In the given figure, $\angle\mathrm{AOB} = 90^\circ$ and $\angle\mathrm{ABC} = 30^\circ$. Then, $\angle\mathrm{CAO} = ?$



(A) 45º (B) 90º (C) 30º (D) 60º

Ans. :

d. 60º

Solution:

We have:

$\angle\mathrm{AOB} = 2\angle\mathrm{ACB}$

$\Rightarrow \angle\mathrm{ACB} = \frac{1}{2}\angle\mathrm{AOB} = \left(\frac{1}{2} \times 90^\circ\right) = 45^\circ$

$\Rightarrow \angle\mathrm{ACB} = 45^\circ$

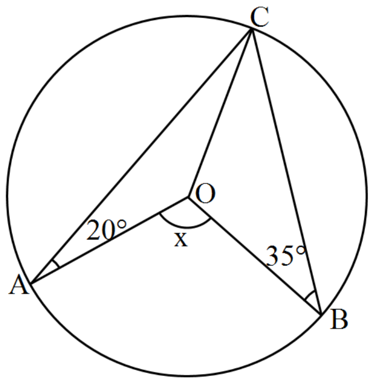
$\angle\mathrm{COA} = 2\angle\mathrm{CBA} = (2 \times 30^\circ) = 60^\circ$

$\therefore \angle\mathrm{COD} = 180^\circ - \angle\mathrm{COA} = (180^\circ - 60^\circ) = 120^\circ$

$\Rightarrow \angle\mathrm{CAO} = \frac{1}{2}\angle\mathrm{COD} = \left(\frac{1}{2} \times 120^\circ\right) = 60^\circ$

$\Rightarrow \angle\mathrm{CAO} = 60^\circ$

95. In the given figure, a circle is centred at O. The value of x is:

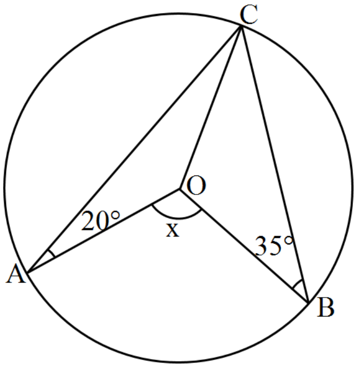


(A) 70º (B) 110º (C) 125º (D) 55º

Ans. :

b. 110º

Solution:



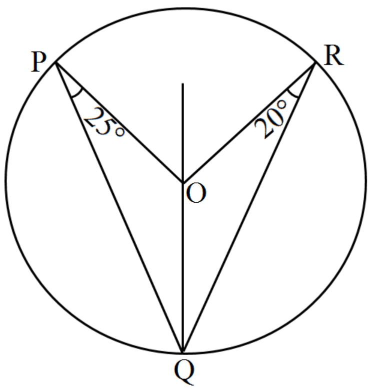
$\angle \mathrm{ACO} = \angle \mathrm{CAO} = 20^\circ$ (because OA = OC)

$\angle \mathrm{OBC} = \angle \mathrm{OCB} = 35^\circ$ (because OB = OC)

$\angle \mathrm{ACB} = 55^\circ$

$\mathrm{x} = 2\angle \mathrm{ACB} = 2 \times 55^\circ = 110^\circ$

96. In the figure, O is the centre of the circle. If $\angle \mathrm{OPQ} = 25^\circ$ and $\angle \mathrm{ORQ} = 20^\circ$, then the measures of $\angle \mathrm{POR}$ and $\angle \mathrm{PQR}$ are respectively:



(A) 90º, 45º (B) 60º, 30º (C) 120º, 60º (D) None of these.

Ans. :

a. 90º, 45º

Solution:

Here, given

OP = OQ and OR = OQ (Radius of circle)

So, {angles opposite to equal sides are also equal}

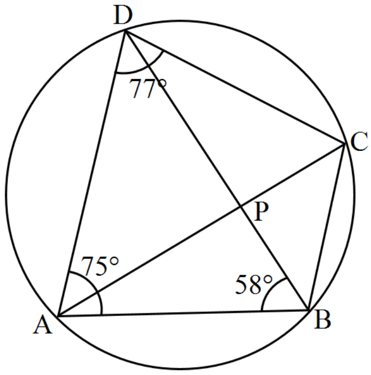
Hence,

PQR = 25º + 20º = 45º

and PQR = 2 PQR = 2 45º = 90º

{Angle subtended by same sides on centre is double the angle at opposite vertex}

97. In the given figure, ABCD is a cyclic quadrilateral in which $\angle \mathrm{BAD} = 75^\circ$, $\angle \mathrm{ABD} = 58^\circ$ and $\angle \mathrm{ADC} = 77^\circ$, AC and BD intersect at P. The measure of $\angle \mathrm{DPC}$ is:

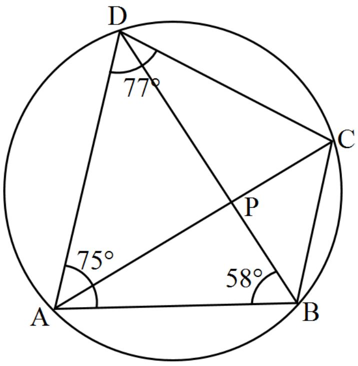


(A) 94º (B) 105º (C) 92º (D) 90º

Ans. :

c. 92º

Solution:



Since AD acts as a chord also, So, $\angle ABD = \angle ACD = 58^\circ$

Again as CD also acts as a chord also, therefore,

$\angle DBC = \angle DAC$

Now, $\angle ABC = \angle ABD + \angle DBC$

Also, $\angle ADC + \angle ABC = 180^\circ$

$\Rightarrow \angle ABC = 180^\circ - 77^\circ = 103^\circ$

And therefore

$\angle DBC = 103^\circ - 58^\circ = 45^\circ$

Hence, $\angle DAC = 45^\circ$

Since,

$\angle DAC = 45^\circ$

So, $\angle CAB = 75^\circ - 45^\circ = 30^\circ$

But, $\angle CAB = \angle BDC$

$\Rightarrow \angle BDC = 30^\circ$

Now, In triangle CPD,

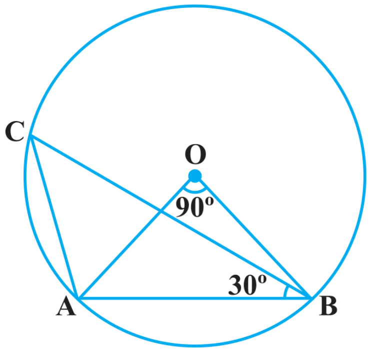
$\angle C + \angle P + \angle D = 180^\circ$

$\Rightarrow 58^\circ + \angle P + 30^\circ = 180^\circ$

$\Rightarrow \angle P = 180^\circ - 30^\circ - 58^\circ = 92^\circ$

98. Write the correct answer in the following:

In Fig. $\angle AOB = 90^\circ$ and $\angle ABC = 30^\circ$, then $\angle CAO$ is equal to:



(A) 30°. (B) 45°. (C) 90°. (D) 60°.

Ans. :

d. 60°.

Solution:

In $\triangle \mathrm{OAB}$, we have

$\mathrm{OA} = \mathrm{OB}$

[Radii of the same circle]

$\therefore \angle \mathrm{OAB} = \angle \mathrm{OBA}$

In triangle OAB, we have

$\angle \mathrm{OAB} + \angle \mathrm{OBA} + \angle \mathrm{AOB} = 180^\circ$

$\therefore 2\angle \mathrm{OAB} = (180^\circ - \angle \mathrm{AOB})$

$= (180^\circ - 90^\circ) = 90^\circ$ [∵ sum of angles of $\triangle$ is 180°]

$\Rightarrow \angle \mathrm{OAB} = \frac{1}{2} \times 90^\circ = 45^\circ$

Also, $\angle \mathrm{ACB} = \frac{1}{2}\angle \mathrm{AOB} = \frac{1}{2} \times 90^\circ = 45^\circ$

Now, in $\triangle \mathrm{CAB}$, we have

$\angle \mathrm{CAB} = 180^\circ - (\angle \mathrm{ABC} + \angle \mathrm{ACB})$

$= 180^\circ - (30^\circ + 45^\circ) = 105^\circ$

Now, $\angle \mathrm{CAO} = \angle \mathrm{CAB} - \angle \mathrm{OAB}$

$\Rightarrow \angle \mathrm{CAO} = 105^\circ - 45^\circ = 60^\circ$

Hence, (d) is the correct answer.

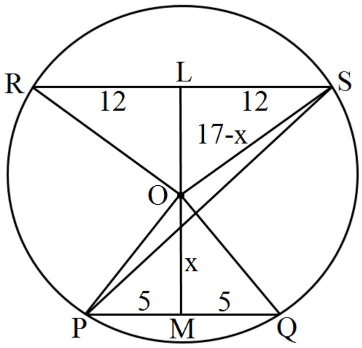
99. PS and RS are two chord's of a circle such that PQ = 10cm and RS = 24cm and PQ || RS. The distance between PQ and RS is 17cm. Find the radius of circle.

(A) 13cm (B) 15cm (C) None of these. (D) 10cm

Ans. :

a. 13cm

Solution:



Let L and M be the midpoints of Rs and PQ respectively.

Let OM = x thus, OL = 17 - x

Now in triangle RLO, RL = 12 and

$RL^2 + OL^2 = r^2$

$12^2 + (17 - x)^2 = r^2$ .....(i)

Similarly,

In triangle OMP, PM = 5 and

$PM^2 + OM^2 = OP^2$

$(x)^2 + 5^2 = r^2$ .....(ii)

Equating (i) and (ii), we get :-

$12^2 + (17 - x)^2 = x^2 + 5^2$

$144 + 289 - 34x + x^2 = x^2 + 25$ {using, $(a - b)^2$}

On solving, we get:

$\mathrm{x} = \frac{408}{34} = 12$

So, 17 - x = 17 - 12 = 5

Thus, from(i),

$RL^2 + OL^2 = r^2$

$12^2 + 5^2 = r^2$

$144 + 25 = r^2$

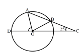
$r^2 = 169$

so, radius $= \sqrt{169}$

Hence, the radius is 13cm.

100. In the given figure, AB is a chord of a circle with centre O and AB is produced to C such that BC = OB. Also, CO is joined and produced to meet the circle in D. If $\angle \mathrm{ACD} = 25^\circ$, then $\angle \mathrm{AOD} = ?$

(A) 50° (B) 75° (C) 90° (D) 100°



Ans. :

b. 75°

Solution:

OB = BC [Given]

$\Rightarrow \angle \mathrm{OBC} = \angle \mathrm{BCO} = 25^\circ$ [Angles opposite equal sides are equal]

Now,

$\angle OBC = \angle BOC + \angle BCO = 25^\circ + 25^\circ = 50^\circ$

OA = OB [Radii of the same circle]

$\Rightarrow \ \angle OAB = \angle OBA = 50^\circ$

In $\triangle AOC,$

$\angle AOD = \angle OAC + \angle ACO$

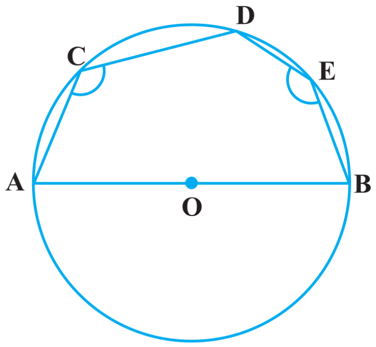
$= \angle OAB + \angle BCO$

$= 50^\circ + 25^\circ$

$= 75^\circ$

* **Answer the following questions. [3 Marks Each]** **[33]**

101. In Fig. AOB is a diameter of the circle and C, D, E are any three points on the semi-circle. Find the value of $\angle ACD + \angle BED$.



Ans. : Join BC.

Since angle in a semicircle is 90°, we have

$\angle ACB = 90^\circ$

As ABCD is a cyclic quadrilateral and opposite angles of a cyclic quadrilateral are supplementary

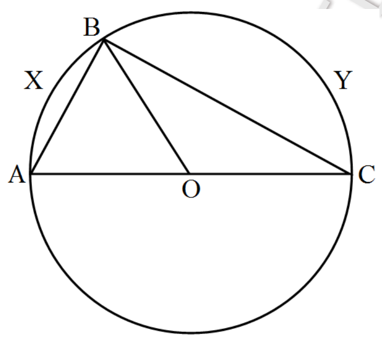
$\therefore \angle BCD + \angle BED = 180^\circ$

Now, adding $\angle ACB$ to both sides, we get

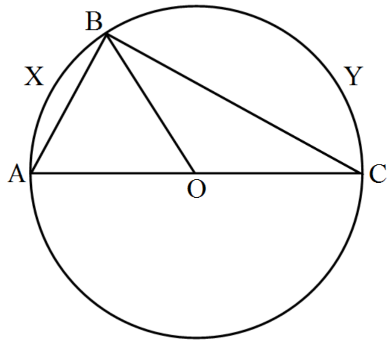
$(\angle BCD + \angle ACB) + \angle BED = 180^\circ + \angle ACB$

Hence, $\angle ACD + \angle BED = 180^\circ + 90^\circ = 270^\circ$

102. If the given figure, AOC is a diameter of the circle and arc $AXB = \frac{1}{2}$ arc BYC. Find $\angle BOC$.



Ans. : We need to find $\angle BOC$



$\text{arc AXB} = \frac{1}{2}\text{arc BYC},$

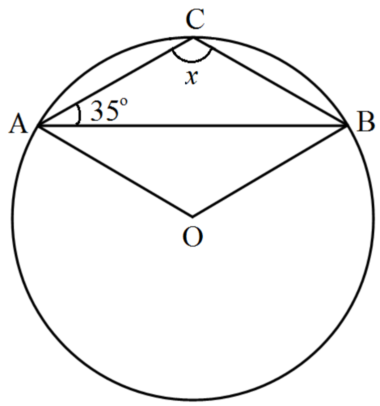
$\angle\text{AOB} = \frac{1}{2}\angle\text{BOC}$

Also $\angle\text{AOB} + \angle\text{BOC} = 180^\circ$

Therefore, $\frac{1}{2}\angle\text{BOC} + \angle\text{BOC} = 180^\circ$

$\Rightarrow \angle\text{BOC} = \frac{2}{3} \times 180^\circ = 120^\circ$

103. If O is the centre of the circle, find the value of x in the following figure:



Ans. : We have

$\angle\text{OAB} = 35^\circ$ then,

$\angle\text{OBA} = \angle\text{OAB} = 35^\circ$ [Angles opposite to equal radii]

In $\triangle\text{AOB},$ by angle sum property

$\Rightarrow \angle\text{AOB} + \angle\text{OAB} + \angle\text{OBA} = 180^\circ$

$\Rightarrow \angle\text{AOB} + 35^\circ + 35^\circ = 180^\circ$

$\Rightarrow \angle\text{AOB} = 180^\circ - 35^\circ - 35^\circ = 110^\circ$

$\therefore \angle\text{AOB} + \text{reflex } \angle\text{AOB} = 360^\circ$ [Complex angle]

$\Rightarrow 110^\circ + \text{reflex } \angle\text{AOB} = 360^\circ$

$\Rightarrow \text{reflex } \angle\text{AOB} = 360^\circ - 110^\circ = 250^\circ$

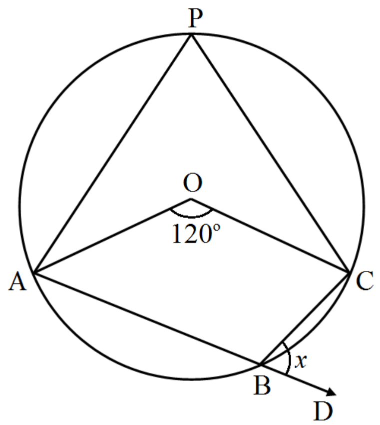
By degree measure theorem reflex

$\angle\text{AOB} = 2\angle\text{ACB}$

$\Rightarrow 250^\circ = 2\text{x}$

$\text{x} = \frac{250^\circ}{2} = 125^\circ$

104. If O is the centre of the circle, find the value of x in the following figure:



Ans. : We have

$\angle AOC = 120^\circ$ By degree measure theorem.

$\angle AOC = 2\angle APC$

$\Rightarrow 120^\circ = 2\angle APC$

$\Rightarrow 120^\circ = 2\angle APC$

$\Rightarrow \angle APC = \frac{120^\circ}{2} = 60^\circ$

$\angle APC + \angle ABC = 180^\circ$ [Opposite angles of cyclic quadrilaterals]

$\Rightarrow 60^\circ + \angle ABC = 180^\circ$

$\Rightarrow \angle ABC = 180^\circ - 60^\circ$

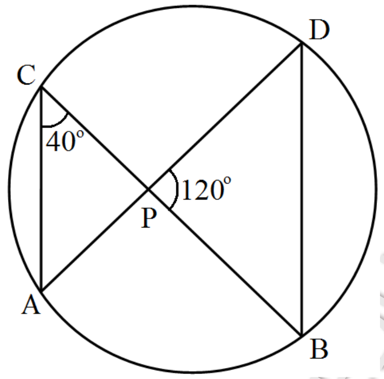
$\Rightarrow \angle ABC = 120^\circ$

$\therefore \angle ABC + \angle DBC = 180^\circ$ [Linear pair of angles]

$\Rightarrow 120^\circ + x = 180^\circ$

$\Rightarrow x = 180^\circ - 120^\circ = 60^\circ$

105. In figure, if $\angle ACB = 40^\circ, \angle DPB = 120^\circ$, find $\angle CBD$.



Ans. : We have,

$\angle ACB = 40^\circ, \angle DPB = 120^\circ$

$\therefore \angle APB = \angle DCB = 40^\circ$ [Angle in same segment]

In $\triangle POB$, by angle sum property

$\angle PDB + \angle PBD + \angle BPD = 180^\circ$

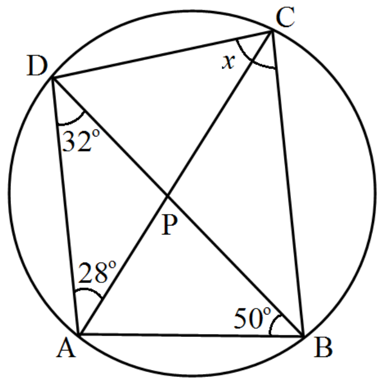
$\Rightarrow 40^\circ + \angle PBD + 120^\circ = 180^\circ$

$\Rightarrow \angle PBD = 180^\circ - 40^\circ - 120^\circ$

$\Rightarrow \angle PBD = 20^\circ$

$\therefore \angle CBD = 20^\circ$

106. If O is the centre of the circle, find the value of x in the following figure:



Ans. : In $\triangle \mathrm{DAB}$, by angle sum property

$\angle \mathrm{ADB} + \angle \mathrm{DAB} + \angle \mathrm{ABD} = 180^\circ$

$\Rightarrow 32^\circ + \angle \mathrm{DAB} + 50^\circ = 180^\circ$

$\Rightarrow \angle \mathrm{DAB} = 180^\circ - 32^\circ - 50^\circ$

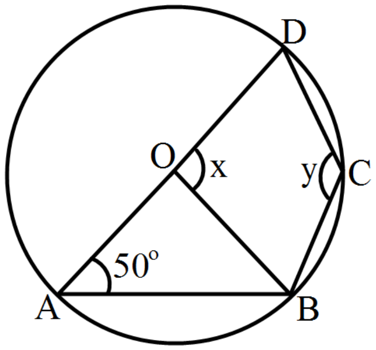
$\Rightarrow \angle \mathrm{DAB} = 98^\circ + 50^\circ$

Now, $\angle \mathrm{OAB} + \angle \mathrm{DCB} = 180^\circ$ [Opposite angles of cyclic quadrilateral]

$\Rightarrow 98^\circ + \mathrm{x} = 180^\circ$

$\Rightarrow \mathrm{x} = 180^\circ - 98^\circ = 82^\circ$

107. In the given figure, O is the center of the circle and $\angle \mathrm{DAB} = 50^\circ$. Calculate the values of x and y.



Ans. : We have, $\angle \mathrm{DAB} = 50^\circ$

By degree measure theorem

$\angle \mathrm{BOD} = 2\angle \mathrm{BAD}$

$\Rightarrow \mathrm{x} = 2 \times 50^\circ = 100^\circ$

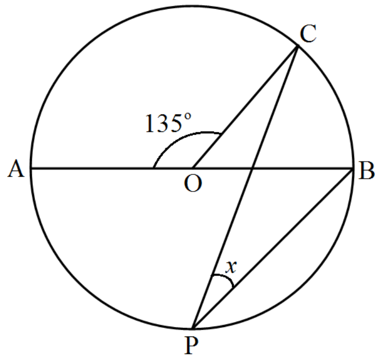
Since, ABCD is a cyclic quadrilateral

Then, $\angle \mathrm{A} + \angle \mathrm{C} = 180^\circ$

$\Rightarrow 50^\circ + \mathrm{y} = 180^\circ$

$\Rightarrow \mathrm{y} = 180^\circ - 50^\circ = 130^\circ$

108. If O is the centre of the circle, find the value of x in the following figures:



Ans. : $\angle AOC = 135^\circ$

$\therefore \angle AOC + \angle BOC = 180^\circ$ [Linear pair of angles]

$\Rightarrow 135^\circ + \angle BOC = 180^\circ$

$\Rightarrow \angle BOC = 180^\circ - 135^\circ$

$\Rightarrow \angle BOC = 45^\circ$

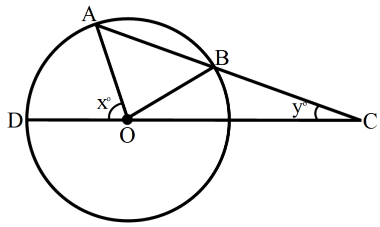
By degree measure theorem

$\angle BOC = 2\angle CPB$

$\Rightarrow 45^\circ = 2x$

$\Rightarrow x = \frac{45^\circ}{2} = 22\frac{1}{2}^\circ$

109. In the given figure, AB is a chord of a circle with centre O and AB is produced to C such that BC = OB. Also, CO is joined and produced to meet the circle in D. If $\angle ACD = y^\circ$ and $\angle AOD = x^\circ$, prove that x = 3y.



Ans. : We have:

$OB = OC, \angle BOC = \angle BCO = y$

External $\angle OBA = \angle BOC + \angle BCO = (2y)$

Again, $OA = OB, \angle OAB = \angle OBA = (2y)$

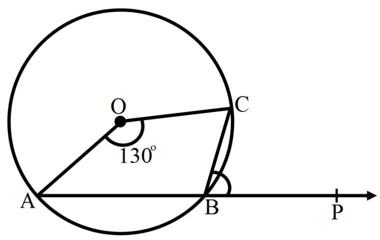
External $\angle AOD = \angle OAC + \angle ACO$

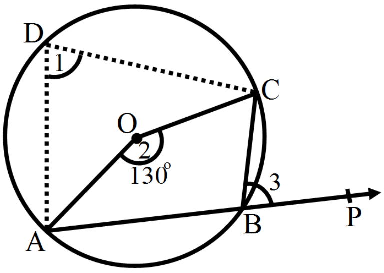
Or $x = \angle OAB + \angle BCO$

Or $x = (2y) + y = 32y$

Hence, $x = 3y$

110. In the given figure, O is the centre of the circle and arc ABC subtends an angle of 130° at the centre. If AB is extended to P, find $\angle PBC$.





Ans. :

Take a points D on the major arc CA and join AD and DC.

$\therefore\ \angle 2 = 2\angle 1$ [Angle subtended by arc is twice the angle subtended by it on the circumference in the alternate segment]

$\therefore\ 130^\circ = 2\angle 1$

$\Rightarrow\ \angle 1 = 65^\circ \ldots (\text{i})$

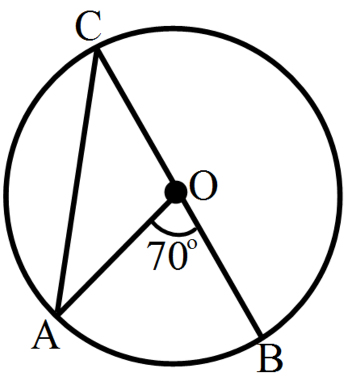
$\angle \text{PBC} = \angle 1$ [$\because$ exterior angle of a cyclic quadrilateral interior opposite angle]

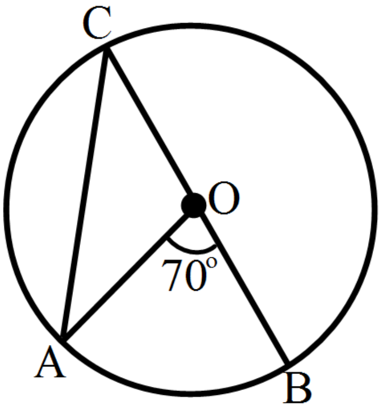
$\therefore\ \angle \text{PBC} = 65^\circ$

111. In the given figure, O is the canter of the circle and $\angle \text{AOB} = 70^\circ$. Calculate the values of

i. $\angle \text{OCA}$

ii. $\angle \text{OAC}$





Ans. :

i. The angle subtended by an arc of a circle at the centre is double the angle subtended by the arc at any point on the circumference.

Thus, $\angle \text{AOB} = 2\angle \text{OCA}$

$\Rightarrow\ \angle \text{OCA} = \left(\frac{\angle \text{AOB}}{2}\right) = \left(\frac{70^\circ}{2}\right)$

$= 35^\circ$

ii. OA = OC [Radii of a circle]

$\angle \text{OAC} = \angle \text{OCA}$ [Base angles of an isosceles triangle are equal]

$= 35^{\circ}$

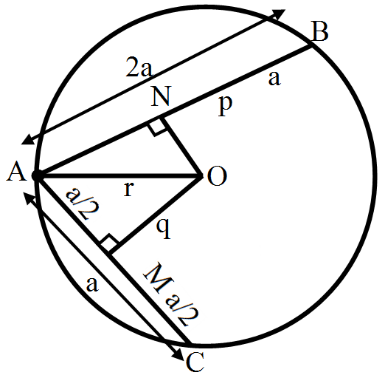
* Questions with calculation. [4 Marks Each] [12]

112. AB and AC are two chords of a circle of radius r such that AB = 2AC. If p and q are the distances of AB and AC from the centre, prove that $4q^2 = p^2 + 3r^2$.

Ans. : Given: In a circlr of radius r, there are two chords AB and AC such that AB = 2AC, Also, the distance of AB and AC from the centre are P and q, respectively.

To prove: $4q^2 + p^2 + 3r^2$,

Proof: Let AC = a, then AB = 2a



From centre O, perpendicular is drawn to the chords AC and AB at M and N, respectively.

$\therefore AM = MC = \frac{a}{2}$

$AN = NB = a$

In $\triangle OAM$, $AO^2 = AM^2 + MO^2$ [by pythagoras theorem]

$\Rightarrow AO^2 = \left(\frac{a}{2}\right)^2 + q^2 \quad \ldots (i)$

In $\triangle OAN$, use pythagoras theorem,

$AO^2 = (AN)^2 + (NO)^2$

$\Rightarrow AO^2 = (a)^2 + p^2 \quad \ldots (ii)$

From Eqs. (i) and (ii),

$\left(\frac{a}{2}\right)^2 + q^2 = a^2 + p^2$

$\Rightarrow \frac{a^2}{4} + q^2 = a^2 + p^2$

$\Rightarrow a^2 + 4q^2 = 4a^2 + 4p^2$ [multiplying both sides by 4]

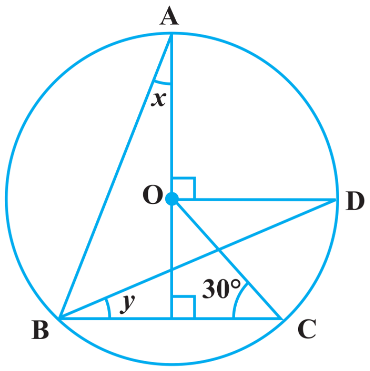
$\Rightarrow 4q^2 = 3a^2 + 4p^2$

$\Rightarrow 4q^2 = p^2 + 3(a^2 + p^2)$

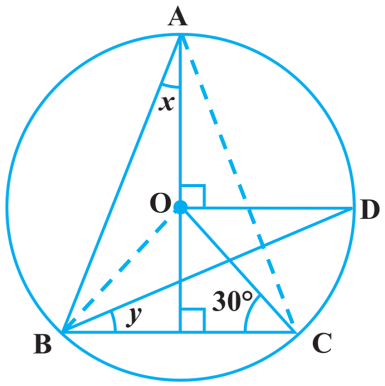
$\Rightarrow 4q^2 = p^2 + 3r^2$ $\left[\text{In right angled } \triangle OAN,\ r^2 = a^2 + p^2\right]$

Hence proved.

113. In Fig. O is the centre of the circle, $\angle BCO = 30^{\circ}$. Find x and y.



Ans. : Given, Ois the centre of the circle and $\angle BCO = 30^\circ$. In the given figure join OB and AC.



In $\triangle BOC, \quad CO = BO$ [both are the radius of circle]

$\therefore \angle OBC = \angle OCB = 30^\circ$ [angles opposite to equal sides are equal]

$\therefore \angle BOC = 180^\circ - (\angle OBC + \angle OCE)$ [by angle sum property of a triangle]

$= 180^\circ - (30^\circ + 30^\circ) = 120^\circ$

$\angle BOC = 2\angle BAC$

We know that, in a circle, the angle subtended by an arc at the centre is twice the angle subtended by it at the remaining part of the circle.

$\therefore \angle BAC = \frac{120^\circ}{2} = 60^\circ$

Also, $\angle BAE = \angle CAE = 30^\circ$ [AE is an angle bisector of angle A]

$\Rightarrow \angle BAE = x = 30^\circ$

In $\triangle ABE, \quad \angle BAE + \angle EBA + \angle AEB = 180^\circ$ [by angle sum property of triangle]

$\Rightarrow 30^\circ + \angle EBA + 90^\circ = 180^\circ$

$\therefore \angle EBA = 180^\circ - (90^\circ + 30^\circ) = 180^\circ - 120^\circ = 60^\circ$

Now, $\angle EBA = 60^\circ$

$\Rightarrow \angle ABD + y = 60^\circ$

$\Rightarrow \frac{1}{2} \times \angle AOD + y = 60^\circ$

[in a circle, the angle subtended by an arc at the centre is twice the angle subtended by it at the remaining part of the circle]

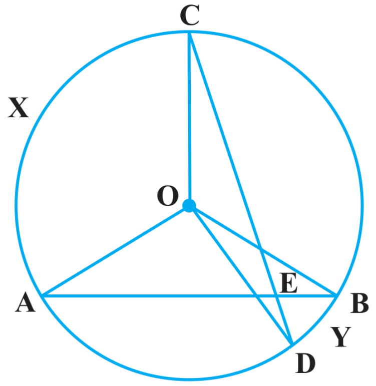
$\Rightarrow \frac{90^\circ}{2} + y = 60^\circ \; [\because \angle AOD = 90^\circ, \text{ given}]$

$\Rightarrow 45^\circ + y = 60^\circ$

$\Rightarrow y = 60^\circ - 45^\circ$

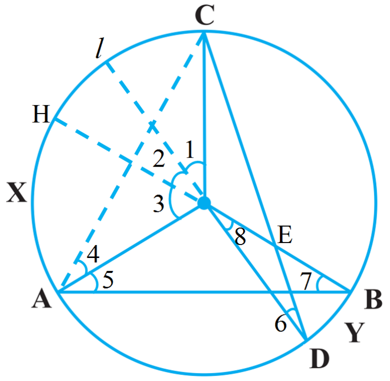
$\therefore y = 15^\circ$

114. In Fig. AB and CD are two chords of a circle intersecting each other at point E. Prove that $\angle AEC = \frac{1}{2}$ (Angle subtended by arc CXA at centre + angle subtended by arc DYB at the centre).



Ans. : Given: In a figure, two chords AB and CD intersecting each other at point E.

To prove: $\angle AEC = \frac{1}{2}$ [angle subtended by arc C × A at centre + angle subtended by arc DYB at the centre]



Construction: Extend the line DO and BO at the points I and H on the circle. Also, join AC.

Proof: We know that, in a circle, the angle subtended by an arc at the centre is twice the angle subtended by it at the remaining part of the circle.

$\therefore \angle 1 = 2\angle 6 \quad \ldots(\text{i})$

and $\angle 3 = 2\angle 7 \quad \ldots(\text{ii})$

In $\triangle AOC, \quad OC = OA$ [both are the radius of circle]

$\angle OCA = \angle 4$ [angles opposite to equal sides are equal]

Also, $\angle AOC + \angle OCA + \angle 4 = 180^\circ$ [by angle sum property of triangle]

$\Rightarrow \angle AOC + \angle 4 + \angle 4 = 180^\circ$

$\Rightarrow \angle AOC = 180^\circ - 2\angle 4 \quad \ldots(\text{iii})$

Now, in $\triangle AEC, \quad \angle AEC + \angle ECA + \angle CAE = 180^\circ$ [by angle property sum of a triangle]

$\Rightarrow \angle AEC = 180^\circ - (\angle ECA + \angle CAE)$

$\Rightarrow \angle AEC = 180^\circ - [(\angle ECO + \angle OCA) + \angle CAO + \angle OAE]$

$= 180^\circ - (\angle 6 + \angle 4 + \angle 4 + \angle 5)$ [In $\triangle OCD, \angle 6 = \angle ECO$ angles opposite to equal sides are equal]

$= 180^\circ - (2\angle 4 + \angle 5 + \angle 6)$

$= 180^\circ - (180^\circ - \angle AOC + \angle 7 + \angle 6)$

[From Eq. (iii) and in $\triangle AOB. \angle 5 = \angle 7,$ as (angles opposite to equal sides are equal)]

$= \angle AOC - \frac{\angle 3}{2} - \frac{\angle 1}{2}$ [from Eqs. (i) and (ii)]

$= \angle AOC - \frac{\angle 1}{2} - \frac{\angle 2}{2} - \frac{\angle 3}{2} + \frac{\angle 2}{2}$ [adding and subtracting $\frac{\angle 2}{2}$]

$= \angle AOC - \frac{1}{2}(\angle 1 + \angle 2 + \angle 3) + \frac{\angle 8}{2}$ [$\because \angle 2 = \angle 8$ (vertically opposite angles)]

$= \angle AOC = \frac{\angle AOC}{2} + \frac{\angle DOB}{2}$

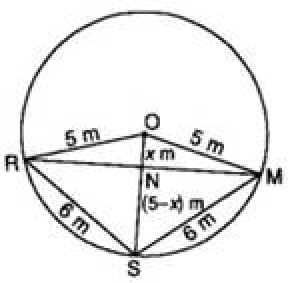
$\Rightarrow \angle AEC = \frac{1}{2}(\angle AOC + \angle DOB)$

$= \frac{1}{2}$ [angle subtended by arc CXA at the centre + angle subtended by arc DYB at the centre]

* Answer the following questions. [5 Marks Each] [65]

115. Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5 m drawn in a park. Reshma throws a ball to Salma. Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6 m each, what is the distance between Reshma and Mandip?

Ans. :



In ΔNOR and ΔNOM

ON = ON |Common

$\angle NOR = \angle NOM$ | $\because$ Equal chords of a circle subtend equal angle at the centre

OR = OM |Radii of a circle

$\therefore \triangle NOR \cong \Delta NOM$ [SAS Rule]

$\therefore \angle ONR = \angle ONM$ [c.p.c.t]

and NR = NM [c.p.c.t.]

But $\angle ONR + \angle ONM = 180^\circ$ |Linear Pair Axiom

$\therefore \angle ONR = \angle ONM = 90^\circ$

$\triangle$ ON is the perpendicular bisector of RM,

Draw bisector SN of $\angle RSM$ to intersect the chord RM in N.

In ΔRSN and ΔMSN

RS = MS (= 6 cm each)

SN = SN [Common]

$\angle RSN = \angle MSN$ [By construction]

$\therefore \Delta RSN \cong \Delta NSN$ [SAS Rule]

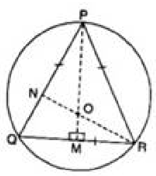
$\therefore \angle RNS = \angle MNS$ [c.p.c.t]

and RN = MN [c.p.c.t]

But $\angle \mathrm{RNS} + \angle \mathrm{MNS} = 180^\circ$ |Linear Pair Axion
$\therefore \angle RNS = \angle MNS = 90^\circ$
$\therefore \mathrm{SN}$ is the perpendicular bisector of RM and therefore passes through O when produced.
Let ON = x m
Then SN = (5 - x) m
In right triangle ONR,
$x^2 + RN^2 = 5^2$, ------ (1) |By Pythagoras theorem
In right triangle SNR,
$(5-x)^2 + RN^2 = 6^2$ ---- (2) |By Pythagoras theorem
From (1),
$RN^2 = 5^2 - x^2$
From (2),
$RN^2 = 6^2 - (5 - x)^2$
Equating the two values of RN^2, we get
$5^2 - x^2 = 6^2 - (5 - x)^2$
$\Rightarrow 25 - x^2 = 36 - (25 - 10x + x)^2 \ \Rightarrow 25 - x^2 = 36 - 25 + 10x - x^2$
$\Rightarrow 25 - x^2 = 11 + 10x - x^2 \Rightarrow 25 - 11 = 10x$
$\Rightarrow$ 14 = 10 x $\Rightarrow$10x = 14
$\Rightarrow x = \frac{14}{10} = 1.4$
Putting x = 1.4 in (1), we get
$(1.4)^2 + RN^2 = 5^2$
$\Rightarrow RN^2 = 5^2 - (1.4)^2 \ \Rightarrow \mathrm{RN}^2 = 25 - 1.96$
$\Rightarrow \mathrm{RN}^2 = 23.04 \Rightarrow \mathrm{RN} = \sqrt{23.04}$
$\Rightarrow$ RN = 4.8
$\therefore$ RM = 2 RN = $2 \times 4.8\mathrm{m} = 9.6\mathrm{m}$
Hence, the distance between Reshma and Mandip is 9.6 m.

116. The circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Ans. : Construction: Draw PM $\perp$ QR and RN $\perp$ PQ
Determination : PQ = QR = RP
$\therefore \triangle$PQR is equilateral. We know that in an equilateral triangle, the medians and the altitudes are the same. So, PM and RN are median. They intersect at O where O is the centre of the circle.



Also, PO = 2 OM = 20 (medians intersect each other in the ratio 2: 1)

$\Rightarrow$ OM = 10 m $\Rightarrow$ PM = OP + OM = 20 + 10 = 30 m

Let QM = x

Then, QM = MR = x [$\because$PM bisects QR]

$\therefore$ QM = $\frac{1}{2}$QR $\Rightarrow$ x = $\frac{1}{2}$QR $\Rightarrow$ QR = 2x

Similarly, PQ = 2x

In right triangle PMQ,

$PQ^2 = PM^2 + QM^2$ |By Pythagoras Theorem

$\Rightarrow (2x)^2 = (30)^2 + x^2$

$\Rightarrow 4x^2 = 900 + x^2$

$\Rightarrow 4x^2 - x^2 = 900$

$\Rightarrow 3x^2 = 900$

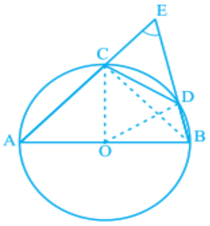
$\Rightarrow x^2 = \frac{900}{3} = 300$

$\Rightarrow x = \sqrt{300} = 10\sqrt{3}$

$\Rightarrow$ PQ = 2x = $2(10\sqrt{3})$

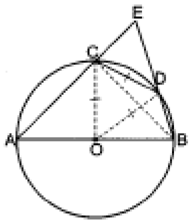
Hence, the length of the string of each phone is $20\sqrt{3}$ m.

117. In given figure, AB is a diameter of the circle, CD is a chord equal to the radius of the circle. AC and BD when extended intersect at point E. Prove that $\angle$AEB = 60°.



Ans. :

Construction: Join OC,OD,BD and BC



Proof:In $\triangle$OCD, we have

OC = OD (Each equal to radius)

and CD=r (given)

So OC=OD=CD

$\therefore$ $\angle$ODC is an equilateral triangle.

$\Rightarrow$ $\angle$COD = 60°

Also, $\angle$COD = 2 $\angle$CBD

$\Rightarrow$ 60° = 2 $\angle$CBD $\Rightarrow$ $\angle$CBD = 30°

Now since $\angle$ACB is angle in a semi-circle.

$\angle$ACB = 90°

$\Rightarrow \angle$BCE = 180° - $\angle$ACB = 180° - 90° = 90°

Thus, in $\triangle$ BCE, we have

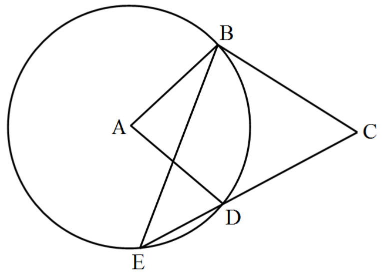
$\angle \mathrm{CEB}+\angle \mathrm{ECB}+\angle \mathrm{CBE}=180^{\circ}$

$\angle \mathrm{CEB}+90^{\circ}+30^{\circ}=180^{\circ}$

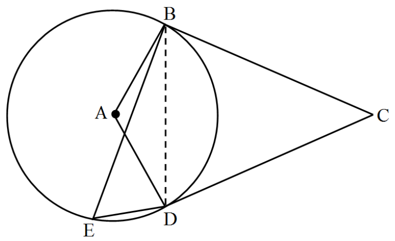
$\angle \mathrm{CEB}=180^{\circ}-120^{\circ}=60^{\circ}$

Hence $\angle \mathrm{CEB}=60^{\circ}$

118. In the given figure, A is the centre of the circle. ABCD is a parallelogram and CDE is a straight line. Find $\angle \mathrm{BCD}$: $\angle \mathrm{ABE}$.



Ans. : It is given that 'ABCD' is a parallelogram. But since 'A' is the centre of the circle, the lengths of 'AB' and 'AD' will both be equal to the radius of the circle.



So, we have AB = AD.

Whenever a parallelogram has two adjacent sides equal then it is a rhombus.

So 'ABCD' is a rhombus.

Let $\angle \mathrm{BDE}=\mathrm{x}^{\circ}$.

We know that in a circle the angle subtended by an arc at the centre of the circle is double the angle subtended by the arc in the remaining part of the circle.

By this property we have

$\angle \mathrm{BAD}=2(\angle \mathrm{BDE})$

$\angle \mathrm{BAD}=2\mathrm{x}^{\circ}$

In a rhombus the opposite angles are always equal to each other.

So, $\angle \mathrm{BAD}=\angle \mathrm{BCE}=2\mathrm{x}^{\circ}$

Since the sum of all the internal angles in any triangle sums up to 180° in triangle $\triangle \mathrm{BEC}$, we have

$\angle \mathrm{BEC}+\angle \mathrm{BCE}+\angle \mathrm{EBC}=180^{\circ}$

$\angle \mathrm{EBC}=180^{\circ}-\angle \mathrm{BEC}-\angle \mathrm{BCE}$

$=180^{\circ}-\mathrm{x}^{\circ}-2\mathrm{x}^{\circ}$

$\angle \mathrm{EBC}=180^{\circ}-3\mathrm{x}^{\circ}$

In the rhombus 'ABCD' since one pair of opposite angles are '2x°' the other pair of opposite angles have to be $\left(180^{\circ}-2\mathrm{x}^{\circ}\right)$

From the figure we see that,

$\angle EBC + \angle AEB = \angle ABC$

$\angle ABE = \angle ABC - \angle EBC$

$180^\circ - 2x^\circ - (180^\circ - 3x^\circ)$

$\angle AEB = x^\circ$

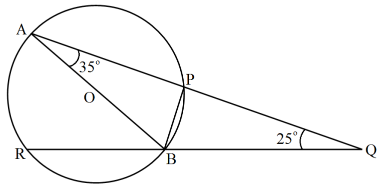
So now we can write the required ratio as,

$\frac{\angle BCD}{\angle ABE} = \frac{2x^\circ}{x^\circ}$

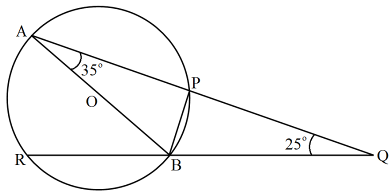
$\frac{\angle BCD}{\angle ABE} = \frac{2}{1}$

Hence the ratio between the given two angles is 2 : 1

119. In the given figure, AB is a diameter of the circle such that $\angle A = 35^\circ$ and $\angle Q = 25^\circ$, find $\angle PBR$.



Ans. : Let us first consider the triangle $\triangle ABQ$.



It is known that in a triangle the sum of all the interior angles add up to 180°.

So here in our triangle $\triangle ABQ$ we have,

$\angle BAQ + \angle AQB + \angle ABQ = 180^\circ$

$\angle ABQ = 180^\circ - \angle BAQ - \angle AQB$

$= 180^\circ - 35^\circ - 25^\circ$

$\angle ABQ = 120^\circ$

By a property of the circle we know that an angle formed in a semi-circle will be 90°.

In the given circle since 'AB' is the diameter of the circle the angle $\angle APB$ which is formed in a semi-circle will have to be 90°.

So, we have $\angle APB = 90^\circ$

Now considering the triangle $\triangle APB$ we have,

$\angle APB + \angle BAP + \angle ABP = 180^\circ$

$\angle APB = 180^\circ - \angle APB - \angle BAP$

$= 180^\circ - 90^\circ - 35^\circ$

$\angle ABP = 55^\circ$

From the given figure it can be seen that,

$\angle ABP + \angle PBQ = \angle ABQ$

$\angle PBQ = \angle ABQ - \angle ABP$

$= 120^\circ - 55^\circ$

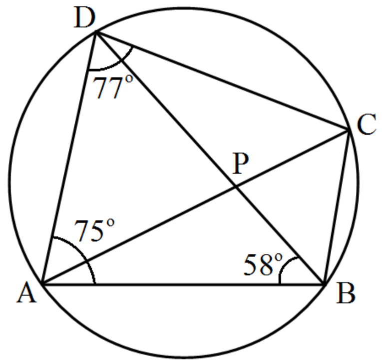
$\angle PBQ = 65^\circ$

Now, we can also say that,

$\angle PBQ + \angle PBR = 180^\circ$
$\angle PBR = 180^\circ - \angle PBQ$
$= 180^\circ - 65^\circ$
$\angle PBR = 115^\circ$

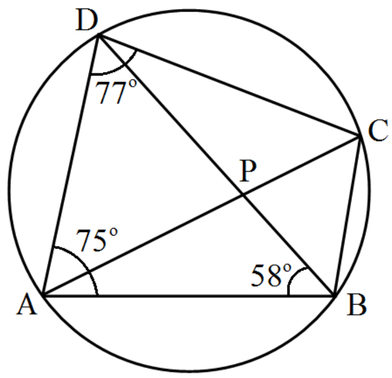
Hence the measure of the angle $\angle PBR$ is 115°.

120. In the given figure, ABCD is a cyclic quadrilateral in which $\angle BAD = 75^\circ,\ \angle ABD = 58^\circ$ and $\angle ADC = 77^\circ$, AC and BD intersect at P. Then, find $\angle DPC$.



Ans. : In a cyclic quadrilateral it is known that the opposite angles are supplementary, meaning that the opposite angles add up to 180°.

Here we have a cyclic quadrilateral ABCD. The centre of this circle is given as ‘O’.



Since in a cyclic quadrilateral the opposite angles are supplementary, here

$\angle ADC + \angle ABD + \angle CBD = 180^\circ$
$\angle CBD = 180^\circ - \angle ADC - \angle ABD$
$= 180^\circ - 77^\circ - 58^\circ$
$\angle CBD = 45^\circ$

Whenever a chord is drawn in a circle two segments are formed. One is called the minor segment while the other is called the major segment. The angle that the chord forms with any point on the circumference of a particular segment is always the same.

Here, ‘CD’ is a chord and ‘A’ and ‘B’ are two points along the circumference on the major segment formed by the chord ‘CD’.

So, $\angle CBD = \angle CAD = 45^\circ$

Now,

$\angle BAD = \angle BAC + \angle CAD$
$\angle BAC = \angle BAD - \angle CAD$
$= 75^\circ - 75^\circ$

$\angle BAC = 30^\circ$

In any triangle the sum of the interior angles need to be equal to 180°.

Consider the triangle $\triangle ABP$,

$\angle PAB + \angle ABP + \angle APB = 180^\circ$

$\Rightarrow \angle APB = 180^\circ - 30^\circ - 58^\circ$

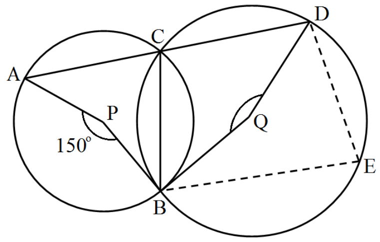
$\Rightarrow \angle APB = 92^\circ$

From the figure, since 'AC' and 'BD' intersect at 'P' we have,

$\angle APB = \angle DPC = 92^\circ$

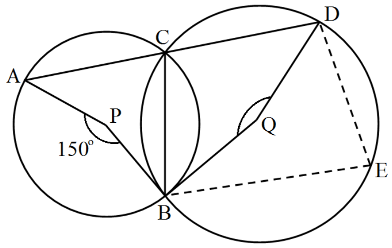
Hence the measure of $\angle DPC$ is 92°.

121. In the given figure, P and Q are centres of two circles intersecting at B and C. ACD is a straight line. Then, $\angle BQD =$



Ans. : Consider the circle with the centre 'P'.

The angle subtended by an arc at the centre of the circle is double the angle subtended by the arc in the remaining part of the circle.



So, here we have

$\angle ACB = \frac{\angle APB}{2}$

$= \frac{150^\circ}{2}$

$\angle ACB = 75^\circ$

Since 'ACD' is a straight line, we have

$\angle ACB + \angle BCD = 180^\circ$

$\angle BCD = 180^\circ - \angle ACB$

$= 180^\circ - 75^\circ$

$\angle BCD = 105^\circ$

Now let us consider the circle with centre 'Q'. Here let 'E' be any point on the circumference along the major arc 'BD'. Now 'CBED' forms a cyclic quadrilateral.

In a cyclic quadrilateral it is known that the opposite angles are supplementary, meaning that the opposite angles add up to 180°.

So here,

$\angle BCD + \angle BED = 180^\circ$

$\angle BED = 180^\circ - \angle BCD$

$= 180^\circ - 105^\circ$

$\angle BED = 75^\circ$

The angle subtended by an arc at the centre of the circle is double the angle subtended by the arc in the remaining part of the circle.

So, now we have

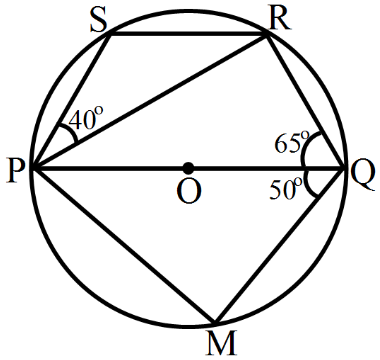
$\angle BQD = 2\angle BED$

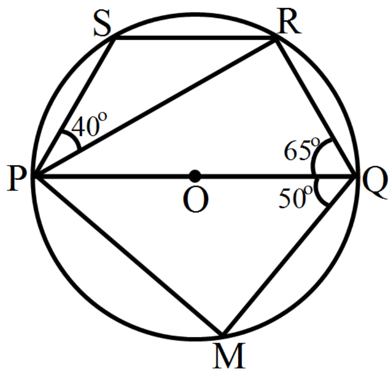
$= 2(75^\circ)$

$\angle BQD = 150^\circ$

Hence, the measure of $\angle BQD$ is 150°.

122. In the given figure, PQ is a diameter of a circle with centre O. If $\angle PQR = 65^\circ, \angle SPR = 40^\circ$ and $\angle PQM = 65^\circ$, find $\angle QPR, \angle QPM$ and $\angle PRS$.





Ans. :

Here, PQ is the diameter and the angle in a semicircle is a right angle.

i.e, $\angle PRQ = 90^\circ$

In $\triangle PRQ$, we have:

$\angle QPR + \angle PRQ + \angle PQR = 180^\circ$ [Angle sum property of a triangle]

$\Rightarrow \ \angle QPR + 90^\circ + 65^\circ = 180^\circ$

$\Rightarrow \ \angle QPR = (180^\circ - 155^\circ) = 25^\circ$

In $\triangle PQM$ PQ is the diameter.

$\angle QPM + \angle PMQ + \angle PQM = 180^\circ$ [Angle sum property of a triangle]

$\Rightarrow \ \angle QPM + 90^\circ + 50^\circ = 180^\circ$

$\Rightarrow \ \angle QPM = (180^\circ - 145^\circ) = 40^\circ$

Now, in quadrilateral PQRS, we have:

$\angle QPS + \angle SRQ = 180^\circ$ [Opposite angles of a cyclic quadrilateral]

$\Rightarrow \ \angle QPR + \angle RPS + \angle PRQ + \angle PRS = 180^\circ$

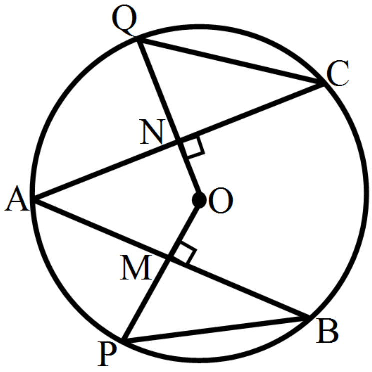
$\Rightarrow \ 25^\circ + 40^\circ + 90^\circ + \angle PRS = 180^\circ$

$\Rightarrow \angle PRS = 180^\circ - 155^\circ = 25^\circ$

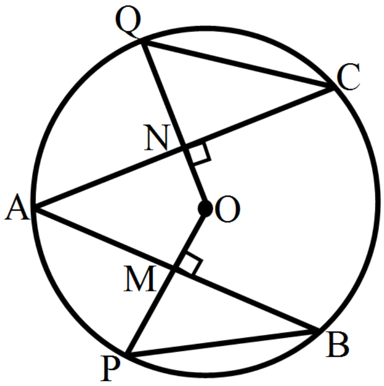
$\therefore \angle PRS = 25^\circ$

Thus, $\angle QPR = 25^\circ, \angle QPM = 25^\circ, \angle PRS = 25^\circ$

123. In the adjoining figure, O is the centre of a circle. If AB and AC are chords of the circle such that AB = AC, $OP \perp AB$ and $OQ \perp AC$,
prove that PB = QC.



Ans. : Given: AB and AC are chords of the circle with centre O. AB = AC, $OP \perp AB$ and $OQ \perp AC$.



To prove: PB = QC

Proof: AB = AC (Given)

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}AC$

The perpendicular from the centre of a circle to a chord bisects the chord.

$\therefore$ MB = NC ...(1)

Also, OM = ON (Equal chords of a circle are equidistant from the centre)

and OP = OQ (Radii)

⇒ OP - OM = OQ - ON

$\therefore$ PM = QN ...(2)

Now, in $\triangle MPB$ and $\triangle NQC$, we have:

MB = NC [From (i)]

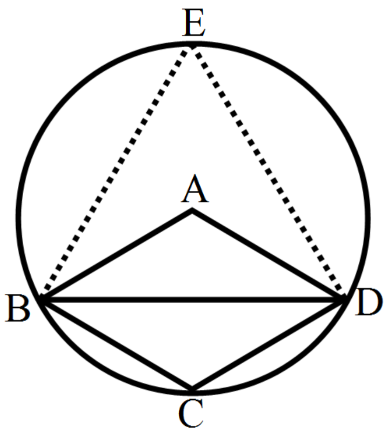
$\angle PMB = \angle QNC$ [90° each]

PM = QN [From (ii)]

i.e., $\triangle MPB \cong \triangle NQC$ (SAS criterion)

$\therefore$ PB = QC (C.P.C.T)

124. ABCD is a quadrilateral such that A is the centre of the circle passing through B, C and D. Prove that $\angle CBD + \angle CDB = \frac{1}{2}\angle BAD$.



Ans. :

Construction: Take a point E on the circle. Join BE, DE and BD.

We know that the angle subtended by an arc of a circle at its centre is twice the angle subtended by the same arc at a point on the circumference.

$\Rightarrow \ \angle \mathrm{BAD} = 2\angle \mathrm{BED}$

$\Rightarrow \ \angle \mathrm{BED} = \frac{1}{2}\angle \mathrm{BAD} \ldots \text{(i)}$

Now, EBCD is a cyclic quadrilateral.

$\Rightarrow \ \angle \mathrm{BED} + \angle \mathrm{BCD} = 180^\circ$

$\Rightarrow \ \angle \mathrm{BCD} = 180^\circ - \angle \mathrm{BED}$

$\Rightarrow \ \angle \mathrm{BCD} = \frac{1}{2}\angle \mathrm{BAD} \ldots \text{(ii)}$ [Using (ii)]

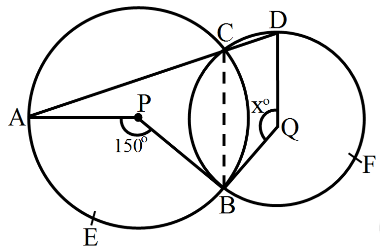
In $\triangle \mathrm{BCD}$, by angle sum property

$\Rightarrow \ \angle \mathrm{CBD} + \angle \mathrm{CDB}\angle \mathrm{BCD} = 180^\circ$

$\Rightarrow \ \angle \mathrm{CBD} + \angle \mathrm{CDB} + 180^\circ - \frac{1}{2}\angle \mathrm{BAD} = 180^\circ$ [Using (ii)]

$\Rightarrow \ \angle \mathrm{CBD} + \angle \mathrm{CDB} = \frac{1}{2}\angle \mathrm{BAD}$

125. In the figure given below, P and Q are centres of two circles, intersecting at B and C, and ACD is a straight line.



If $\angle \mathrm{APB} = 150^\circ$ and $\angle \mathrm{BQD} = \mathrm{x}^\circ$, find the value of x.

Ans. : We know that the angle subtended by an arc of a circle at the centre is double the angle subtended by it on the remaining part of the circle.

Here, arc AEB subtends $\angle \mathrm{APB}$ at the centre and $\angle \mathrm{ACB}$ at C on the circle.

$\therefore \ \angle \mathrm{APB} = 2\angle \mathrm{ACB}$

$\Rightarrow \ \angle \mathrm{ACB} = \frac{150^\circ}{2} = 75^\circ \ldots (1)$

Since ACD is a straight line, $\angle \mathrm{ACB} = 2\angle \mathrm{BCD} = 180^\circ$

$\Rightarrow \ \angle \mathrm{BCD} = 180^\circ - 75^\circ$

$\Rightarrow \ \angle \mathrm{BCD} = 105^\circ \ldots (2)$

Also, arc BFD subtends reflex $\angle \mathrm{BQD}$ at the centre and $\angle \mathrm{BCD}$ at C on the circle.

$\therefore$ reflex $\angle \mathrm{BQD} = 2\angle \mathrm{BCD}$

$\Rightarrow$ reflex $\angle BQD = 2(105^\circ) = 210^\circ \ldots (3)$

Now,

reflex $\angle BQD + \angle BQD = 360^\circ$

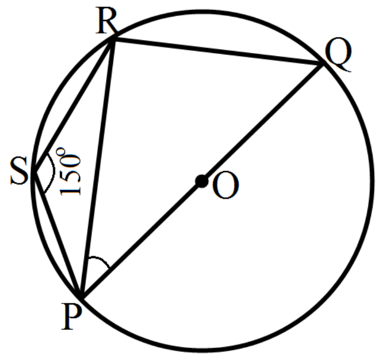
$\Rightarrow \ 210^\circ + x = 360^\circ$

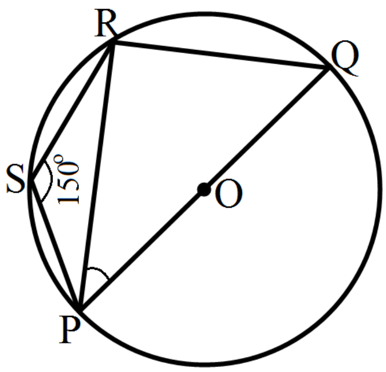
$\Rightarrow \ x = 360^\circ - 210^\circ$

$\Rightarrow \ x = 150^\circ$

Hence, $x = 150^\circ$.

126. In the given figure, POQ is a diameter and PQRS is a cyclic quadrilateral. If $\angle PSR = 150^\circ$, find $\angle RPQ$.





Ans. :

In cyclic quadrilateral PQRS, we have:

$\angle PSR + \angle PQR = 180^\circ$

$\Rightarrow \ 150^\circ + \angle PQR = 180^\circ$

$\Rightarrow \ \angle PQR = (180^\circ - 150^\circ) = 30^\circ$

$\therefore \ \angle PQR = 30^\circ \ldots (i)$

Also, $\angle PQR = 90^\circ \ldots (ii)$ (Angle in a semicircle)

Now, in $\triangle PRQ$, we have:

$\angle PQR + \angle PRQ + \angle RPQ = 180^\circ$

$\Rightarrow \ 30^\circ + 90^\circ + \angle RPQ = 180^\circ$ [From (i) and (ii)]

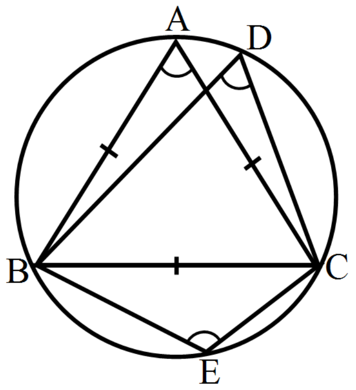
$\Rightarrow \ \angle RPQ = 180^\circ - 120^\circ = 60^\circ$

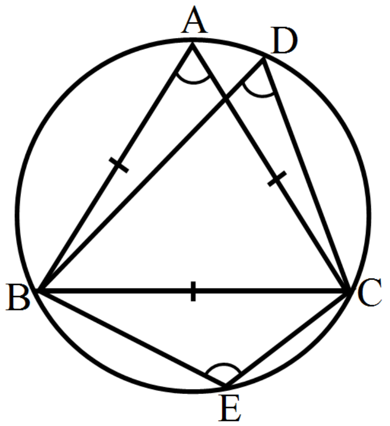
$\therefore \ \angle RPQ = 60^\circ$

127. In the given figure, $\triangle ABC$ is equilateral. Find

i. $\angle BDC$

ii. $\angle BEC$





Ans. :

i. Given: $\triangle \mathrm{ABC}$ is an equilateral triangle

i.e., each of its angle = 60°

$\Rightarrow \angle \mathrm{BAC}=\angle \mathrm{ABC}=\angle \mathrm{ACB}=60^{\circ}$

Angles in the same segment of a circle are equal.

$\text{i.e.,} \angle \mathrm{BDC}=\angle \mathrm{BAC}=60^{\circ}$

$\therefore \angle \mathrm{BDC}=60^{\circ}$

ii. The opposite angles of a cyclic quadrilateral are supplementary.

Then in cyclic quadrilateral ABEC, we have:

$\angle \mathrm{BAC}+\angle \mathrm{BEC}=180^{\circ}$

$\Rightarrow 60^{\circ}+\angle \mathrm{BEC}=180^{\circ}$

$\Rightarrow \angle \mathrm{BEC}=\left(180^{\circ}-60^{\circ}\right)=120^{\circ}$

$\therefore \angle \mathrm{BDC}=60^{\circ}$ and $\angle \mathrm{BEC}=120^{\circ}$

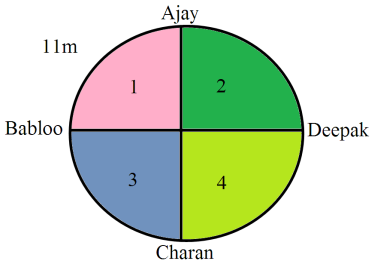
*** Case study based questions. [20]**

128. Read the Source/ Text given below and answer any four questions:

Four students of class IX B with names Ajay, Babloo, Charan and Deepak are playing a game in a circular playground.

All four students are holding radios with speaker and mic. These radios are connected by a wire of equal length that is 11m (for each radio). Ajay Asks a question to Babloo. If Babloo gives the correct answer he gets 10 points and asks a new question to Charan, If he can not answer then he passes the same question to Charan and gets no points.

These conditions apply to all four players. After 10 rounds who gets maximum points, he becomes the winner.

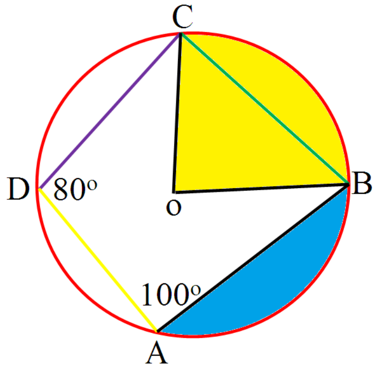


i. What is the radius of the field?
   a. 7m
   b. 14m
   c. 11m
   d. 22m
ii. What is the area of the field?
   a. $70m^2$
   b. $154m^2$
   c. $110m^2$
   d. $220m^2$
iii. What is the area of the part marked with 1 on the field?
   a. $50m^2$
   b. $154m^2$
   c. $76m^2$
   d. $38.5m^2$
iv. What is the circumference of the field?
   a. 22m
   b. 14m
   c. 44m
   d. 28m
v. What is the direct distance from Ajay to Charan?
   a. 7m
   b. 28m
   c. 15m
   d. 14m

Ans. :

i	a	7m
ii	b	$154m^2$
iii	d	$38.5m^2$
iv	c	44m
v	d	14m

129. Read the Source/ Text given below and answer these questions:



There was a circular park in Defence colony At Delhi. For fencing purpose Poles A, B, C and D were installed at the circumference of the park. Ram tied wires From A to B to C and C to D, He managed to measure the $\angle A = 100^\circ$ and $\angle D = 80^\circ$ The point O in the middle of the park is the center of the circle.

Now answer the following questions:

i. What is the value of $\angle B$?
   a. 80°
   b. 100°
   c. 90°
   d. 70°

ii. What is the value of $\angle C$?
   a. 80°
   b. 100°
   c. 90°
   d. 70°

iii. What is the special type of quadrilateral ABCD?
   a. Square.
   b. Rectangle.
   c. Cyclic quadrilateral.
   d. Trapezium.

iv. What is the property of cyclic quadrilateral?
   a. Opposite angles are supplementary.
   b. Adjacent angles are equal.
   c. Opposite angles are equal.
   d. Adjacent angles are complementary.

v. What you will call the yellow shaded shape OBC?
   a. Segment.
   b. Arc.
   c. Chord.
   d. Sector.

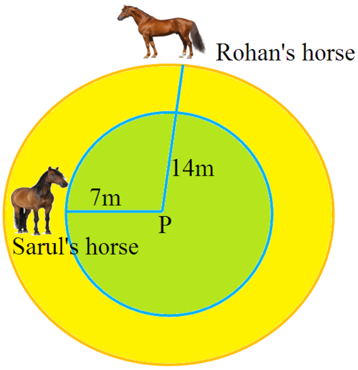
Ans. :

(i)	(b)	100°
(ii)	(a)	80°
(iii)	(c)	Cyclic quadrilateral.
(iv)	(a)	Opposite angles are supplementary.
(v)	(d)	Sector.

130. Read the Source/ Text given below and answer any four questions:

Rohan and Suraj were close friends, One day they were riding horses from Delhi to Faridabad. The names of their horses were Saku and Fareed respectively. The day was very sunny. On the way, they stopped for resting in a park. They tied their horses to a tree in the park. The length of ropes of Rohans's horse is 14m and that of the horse of Suraj is 7m as shown in the figures.

Both the friends slept in the park under a green tree for some time. During this period both the horses took 10 rounds along with the tree they were tied.



Answer the following questions

i. The ratio of distance walked in 10 rounds by the horses of Rohan and Suraj is:
   a. 2 : 1
   b. 1 : 2
   c. 3 : 1
   d. 1 : 3

ii. The ratio of area of the grass the horses of Rohan and Suraj could graze:
   a. 2 : 1
   b. 1 : 2
   c. 4 : 1
   d. 1 : 4

iii. What is the distance walked by Rohan's horse in 5 rounds:
   a. 220m
   b. 100m
   c. 440m
   d. 110m

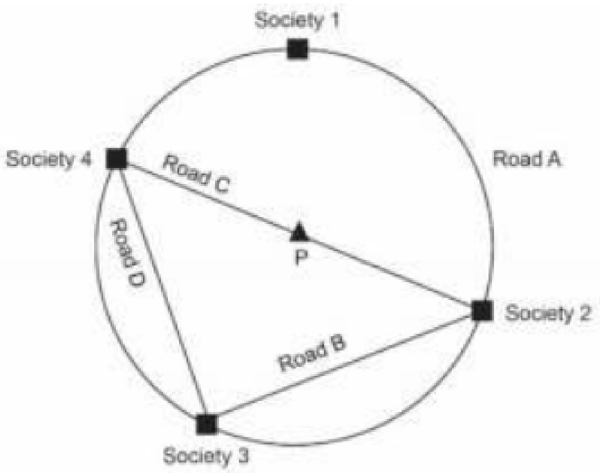
iv. What we call the the length of rope in terms of circle?
   a. Diameter
   b. Radius
   c. Chord
   d. Tangent

v. What we call the the distance walked by a horse in one round?
   a. Area
   b. Radius
   c. Circumference
   d. diameter

Ans. :

i	a	2 : 1
ii	c	4 : 1
iii	c	440m
iv	b	Radius
v	c	Circumference

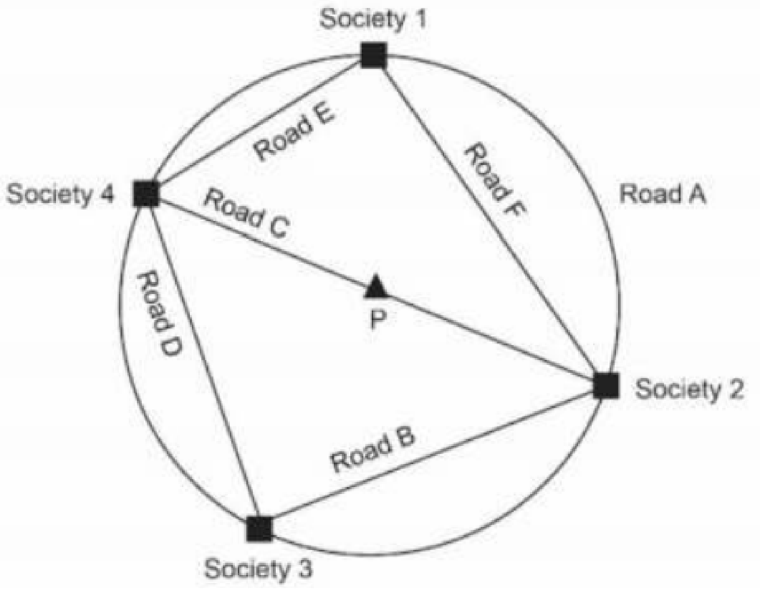
131. Given below is the map giving the position of four housing societies in a township connected by a circular road A.



Society 2 and 3 are connected by straight road B, society 4 and 2 are connected by straight road C and society 4 and 3 are connected by road D. Point P denotes the position of a park. The park is equidistant to all four societies.
Rubina claims that it is not possible to construct another circular road connecting all four societies.
1. Which of the following options justiies Rubina’s claim?
  A. Equal chords of congruent circles subtend equal angles at the centre.
  B. The perpendicular from the centre of a circle to a chord bisects the chord.
  C. There is a unique circle passing through three non-collinear points.
  D. Points equidistant from a given point will lie on a circle.
2 What is the position of the park P with respect to road A?
  A. Chord
  B. Centre
  C. Sector
  D. Segment
3. The length of Road B is equal to the length of Road D.
Which of the following options can be true for the roads in the township?
  A. Road B bisects Road D.
  B. Road B and Road make an acute angle.
  C. Road B, Road C and Road D are of equal length.
  D. Road B and Road D subtend equal angles at society 1.
4. Alex says, “The angle made by road B on road D is a right angle.”
Jai and Angad give different justiications to support Alex’s claim.
Jai says, “Angles in the same segment of a circle are equal.”
Angad says, “The angle in a semicircle is a right angle.”
Who has given the correct justiication?

Ans. : 1. C. There is a unique circle passing through three non-collinear points.
2. B. Centre
3. D. Road B and Road D subtend equal angles at society 1.
4. Angad is correct.

132. Two new roads, Road E and Road F were constructed between society 4 and 1 and society 1 and 2.



5. What would be the measure of the sum of angles formed by the straight roads at society 1 and society 3?
   A. 60°
   B. 90°
   C. 180°
   D. 360°

6. Krish says, "The distance to go from society 4 to society 2 using Road D will be longer that the distance using Road E"
Is Krish correct? Justify your answer with examples.

7. Road G, perpendicular to Road F was constructed to connect the park and Road F. Which of the following is true for Road G and Road F?
   A. Road G and road F are of same length.
   B. Road F divides Road G into two equal parts.
   C. Road G divides Road F into two equal parts.
   D. The length of road G is one-fourth of the length of Road F.

8. Priya said, "Minor arc corresponding to Road B is congruent to minor arc corresponding to Road D."
Do you agree with Priya? Give reason to support your answer.

Ans. : 5. C. 180°
6. Examples to show that in a right triangle the sum of legs is longest for an isosceles right triangle when hypotenuse remains same.
● Take for example the length of diameter (hypotenuse) = 5 units.
Road D and Road B are equal hence (Road D = 3.53 units).
Let Road E be = 1 Chapter, Road F = 4.89 units.
Therefore, length of Road B + Road D is greater than Road E + Road F.
7. C. Road G divides Road F into two equal parts.
8. Yes, Priya is correct with valid reasoning.
● Yes, Priya is correct because arc corresponding to two equal roads (chords) are congruent.

----- "Our greatest glory is not in never falling, but in rising every time we fall." -----